

Fractals in Seeded TA

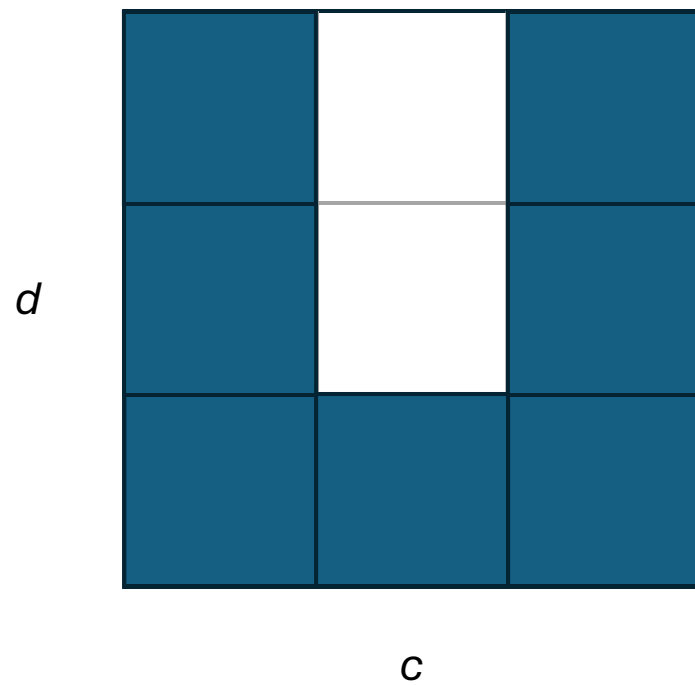


Authors: Asher Haun, **Ryan Knobel**, Adrian Salinas, Ramiro Santos,
Robert Schweller, Timothy Wylie

Discrete Self-Similar Fractals

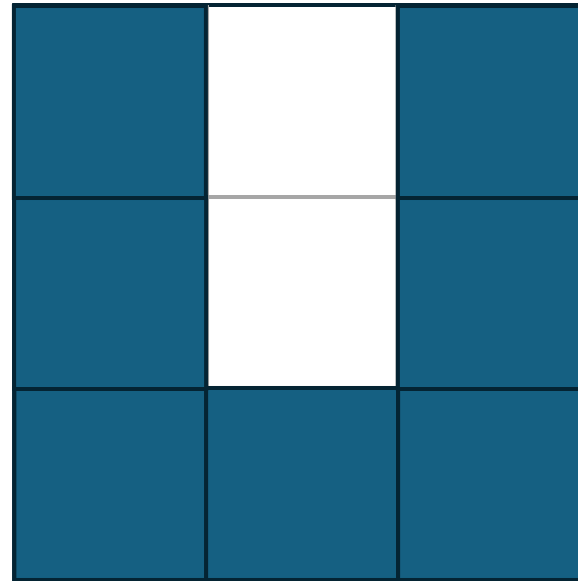
What are they?

Generator



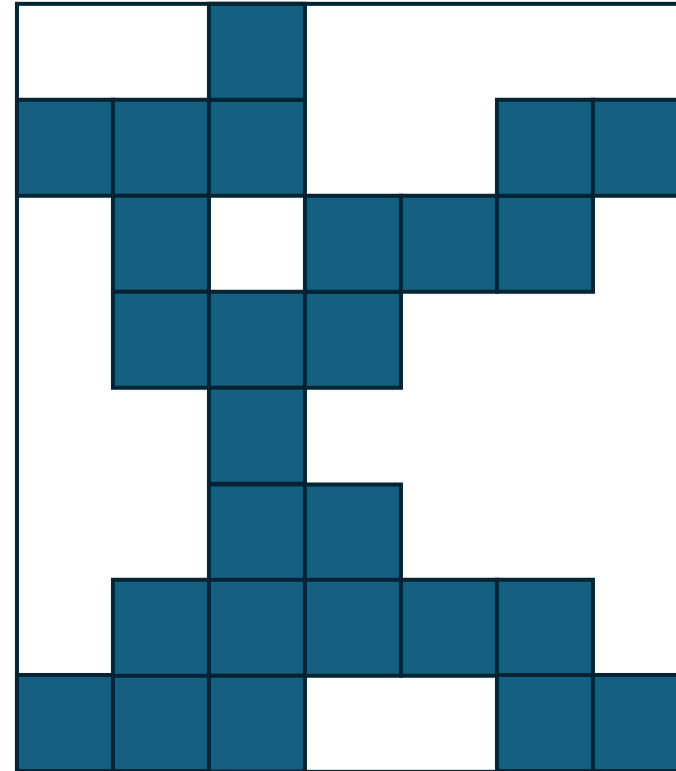
Generator

1. Connected



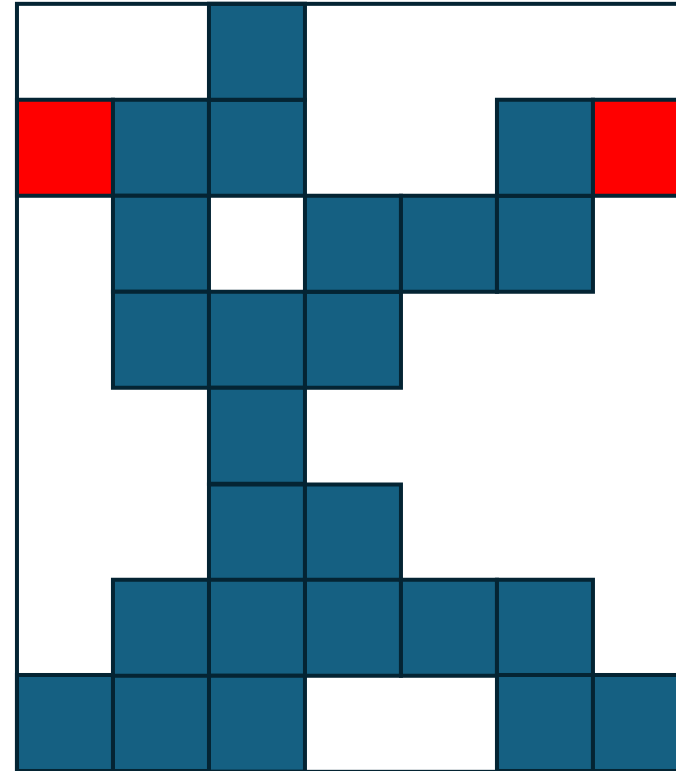
Generator

1. Connected
2. Feasible



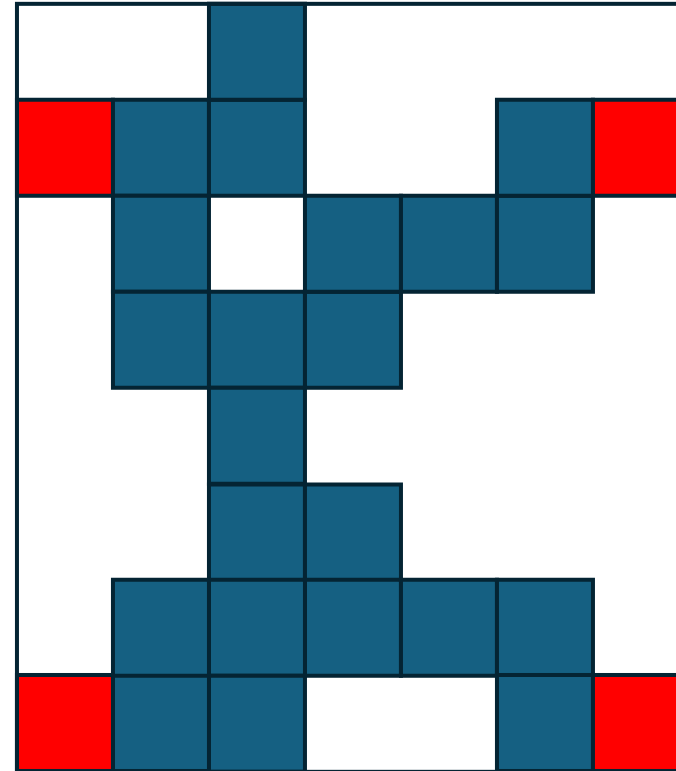
Generator

1. Connected
2. Feasible



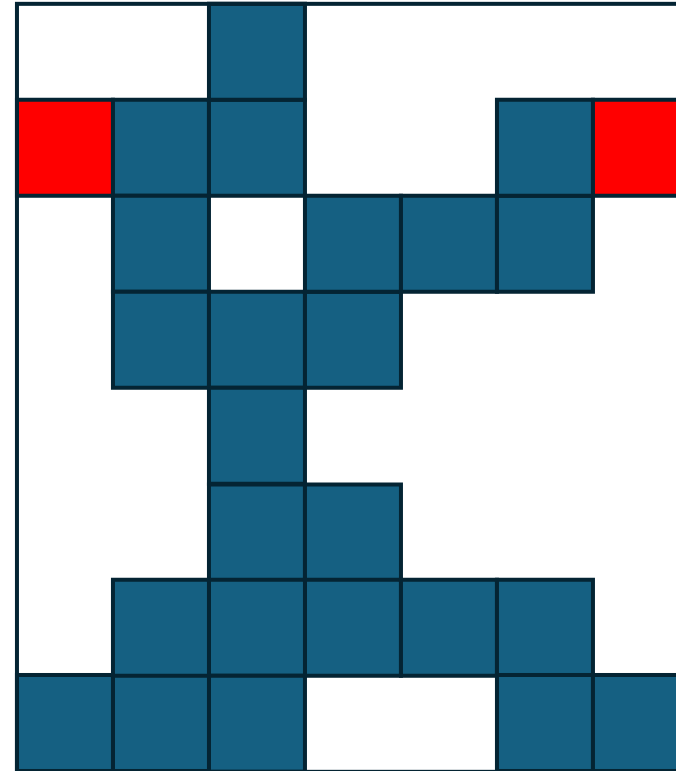
Generator

1. Connected
2. Feasible



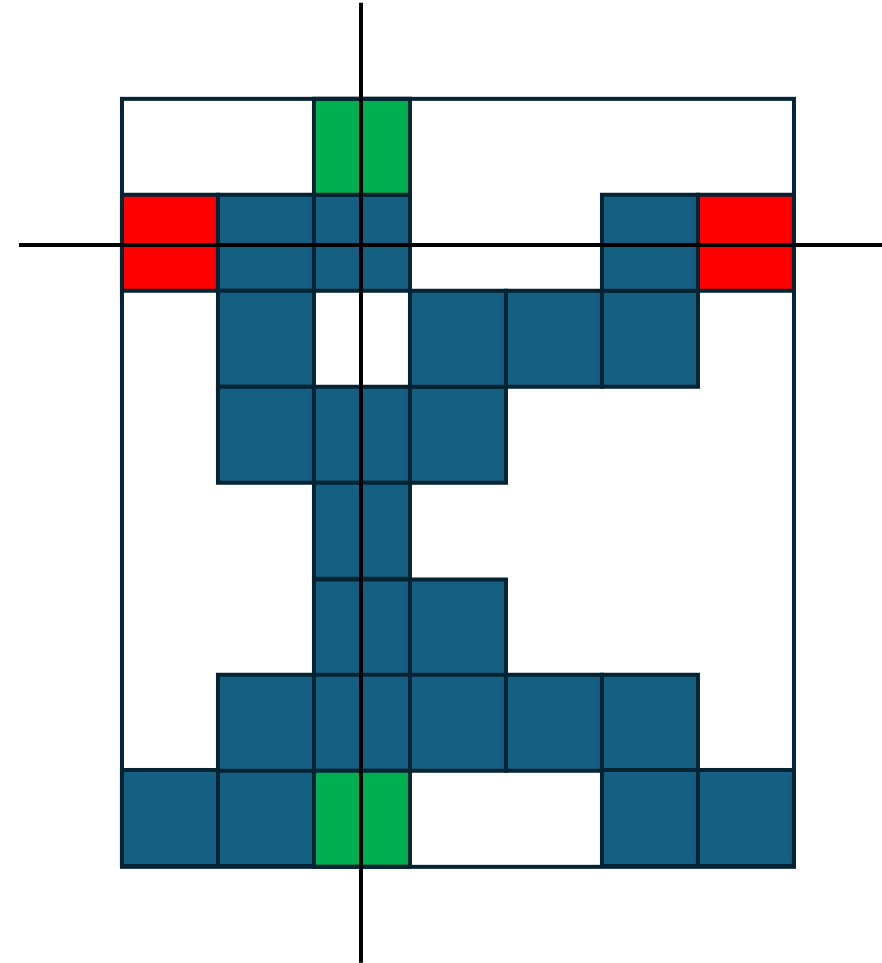
Generator

1. Connected
2. Feasible



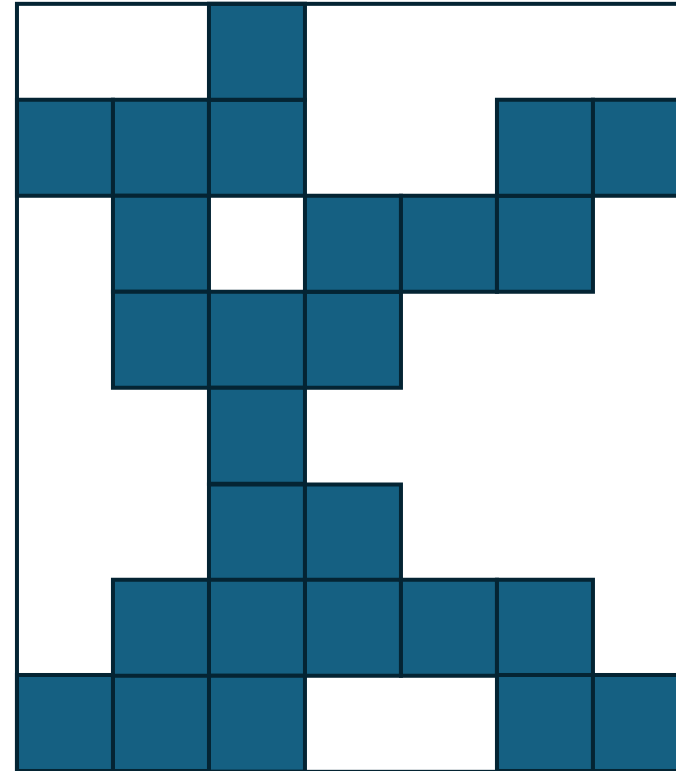
Generator

1. Connected
2. Feasible



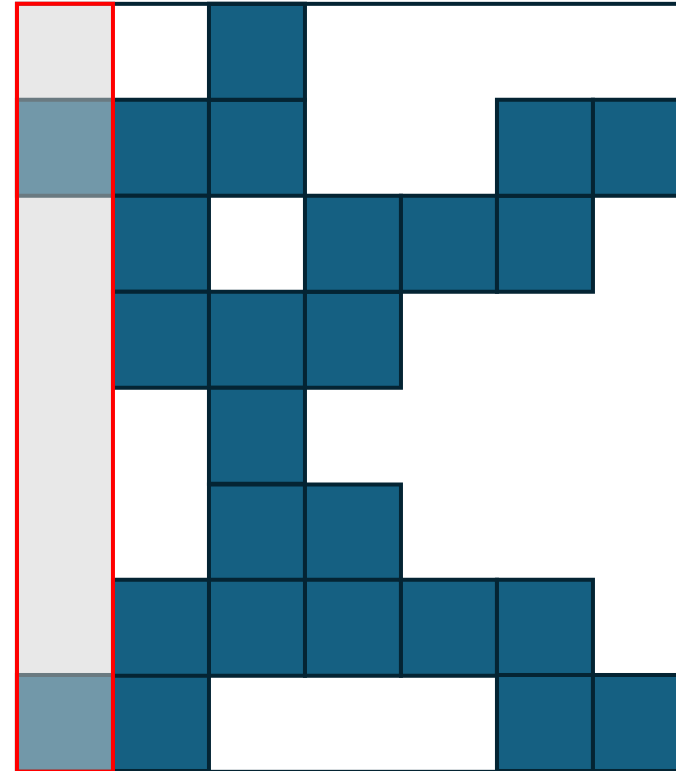
Generator

1. Connected
2. Feasible



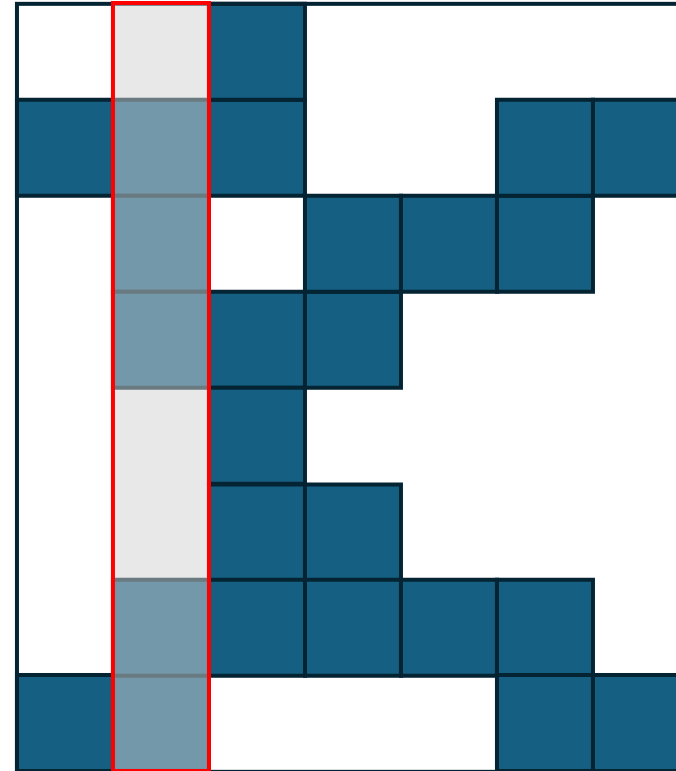
Generator

1. Connected
2. Feasible



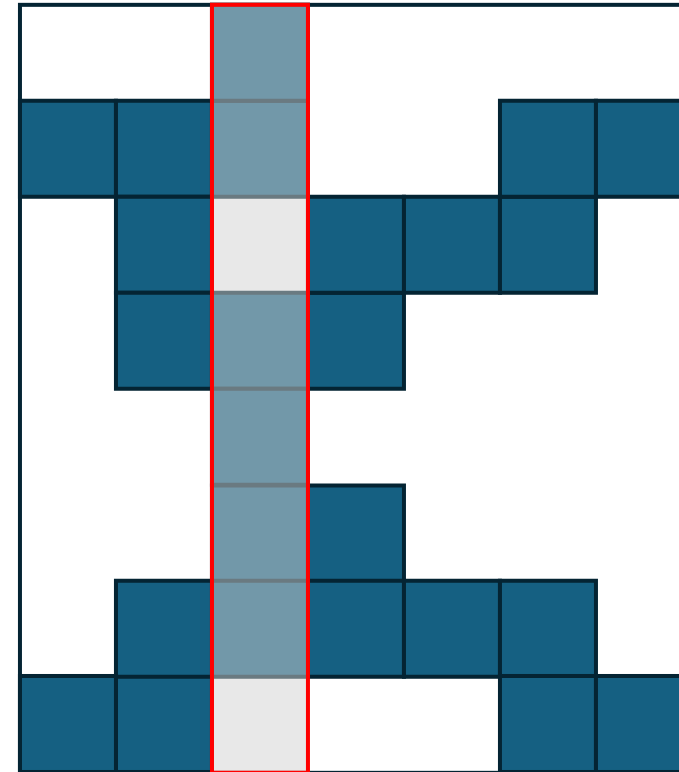
Generator

1. Connected
2. Feasible



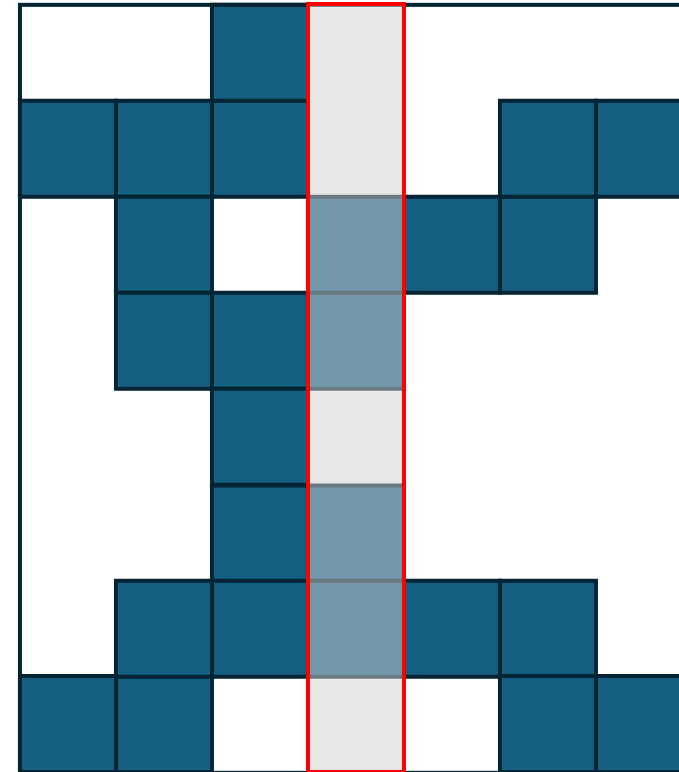
Generator

1. Connected
2. Feasible



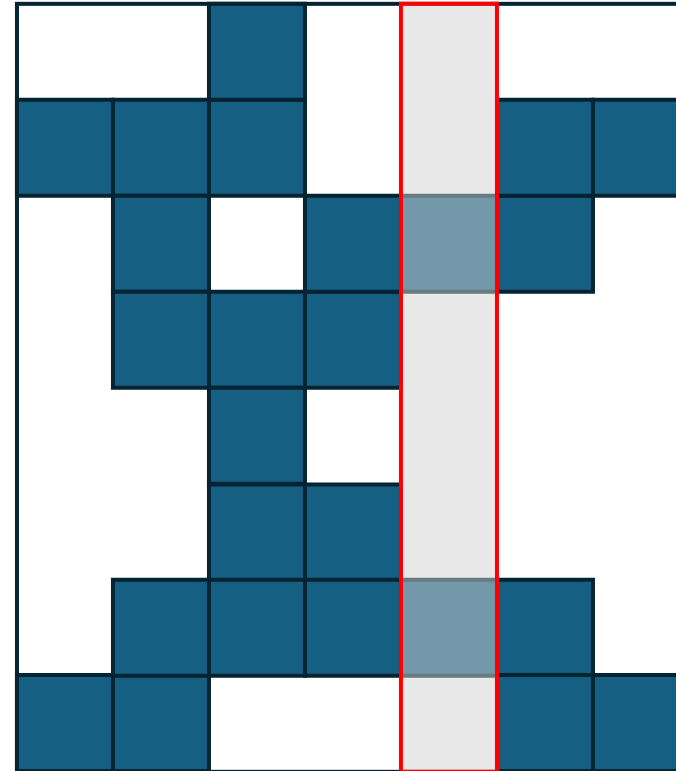
Generator

1. Connected
2. Feasible



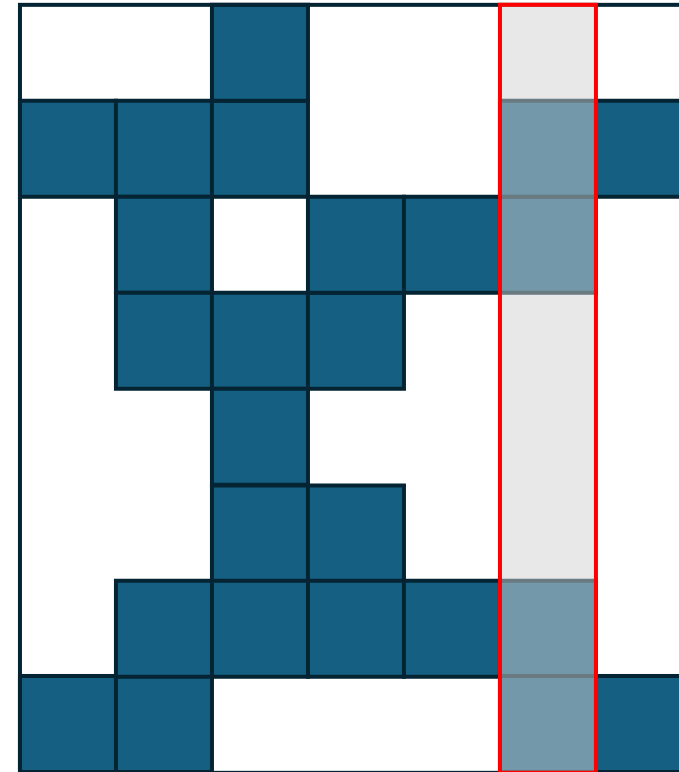
Generator

1. Connected
2. Feasible



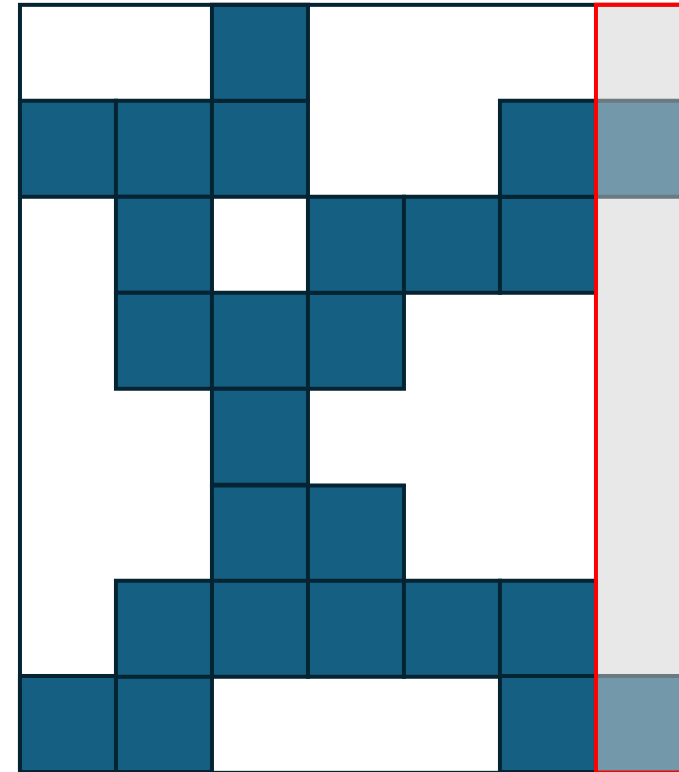
Generator

1. Connected
2. Feasible



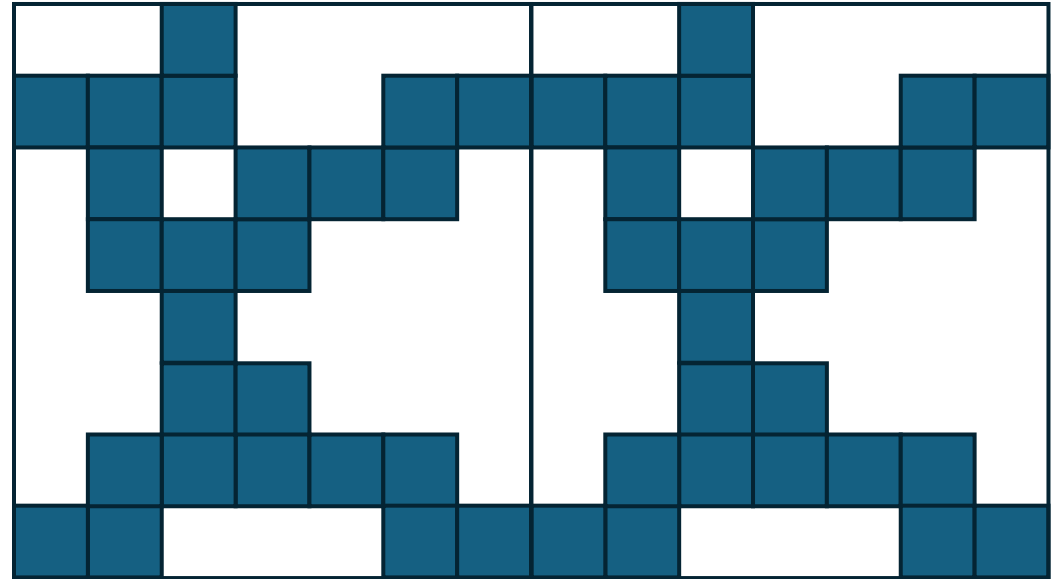
Generator

1. Connected
2. Feasible



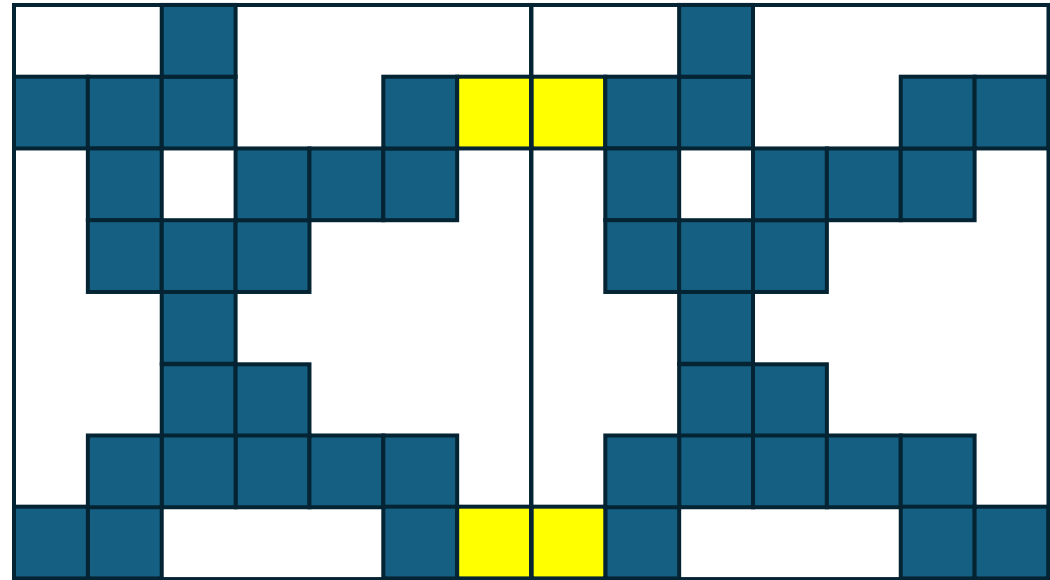
Generator

1. Connected
2. Feasible



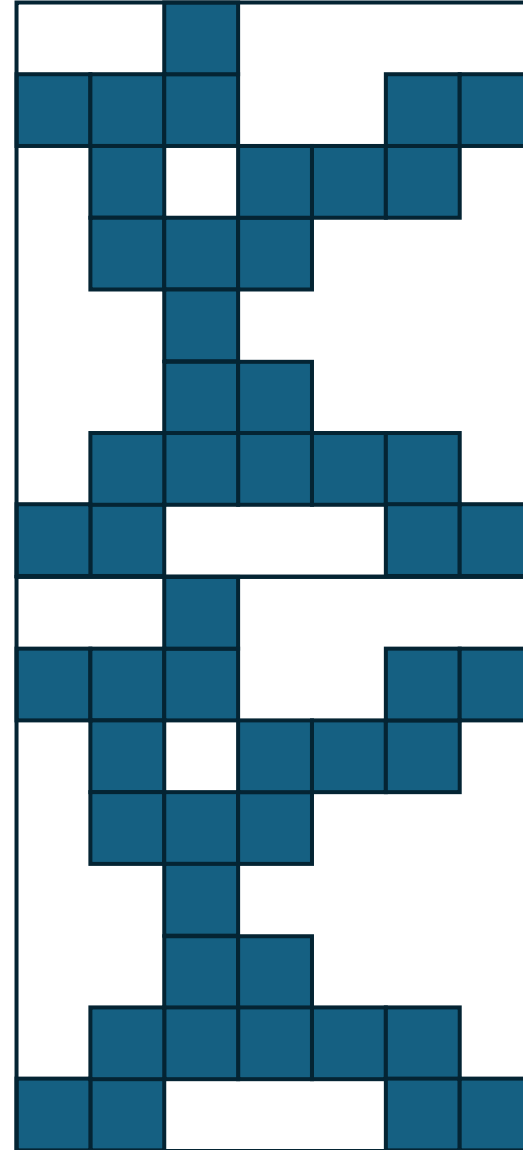
Generator

1. Connected
2. Feasible

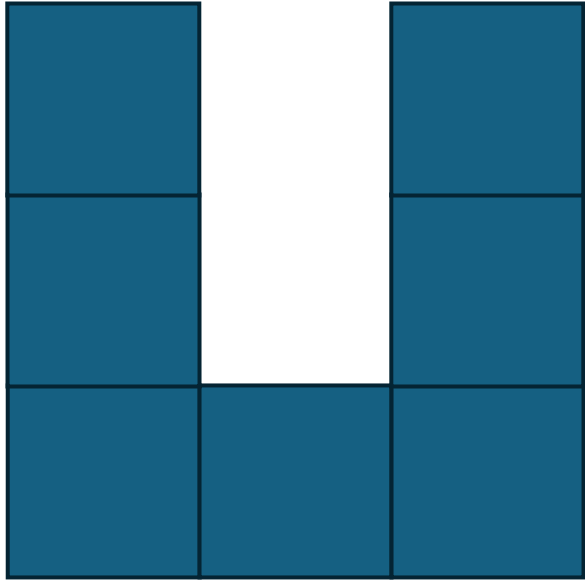


Generator

1. Connected
2. Feasible

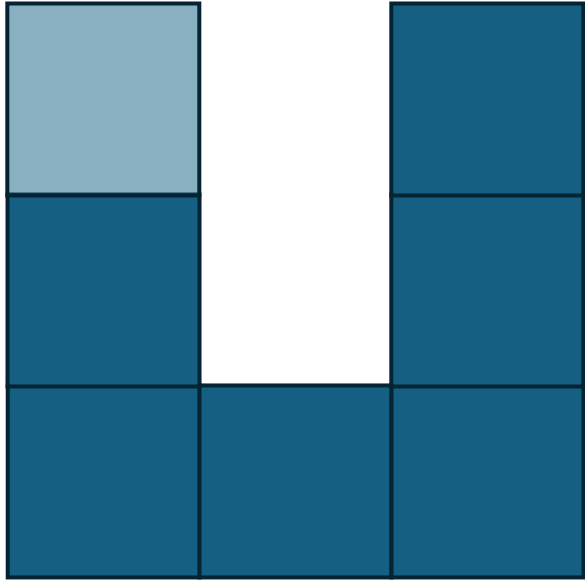


Discrete Self-Similar Fractal

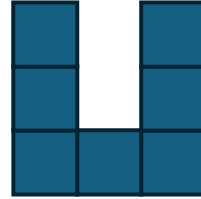


Stage 1

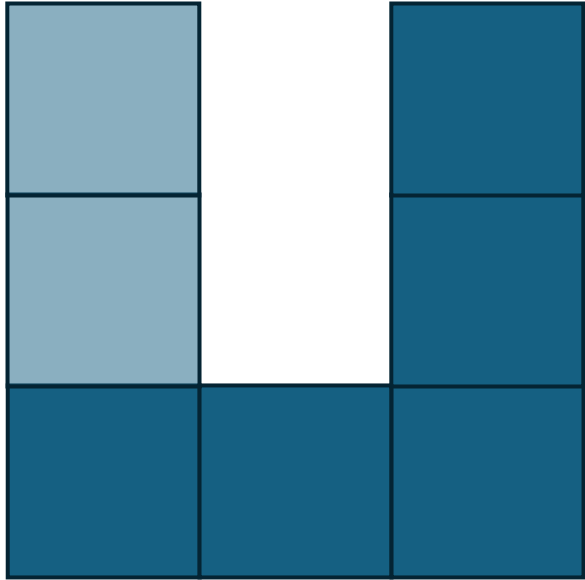
Discrete Self-Similar Fractal



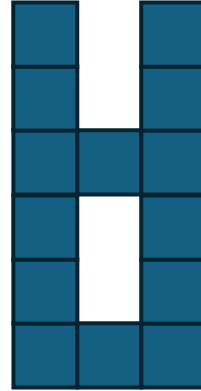
Stage 1



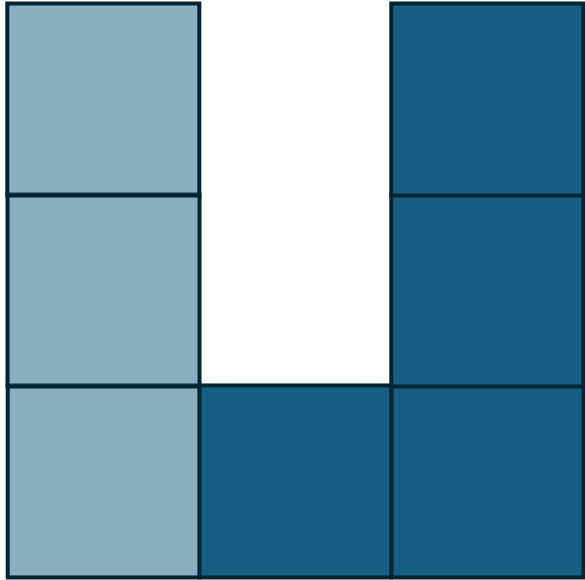
Discrete Self-Similar Fractal



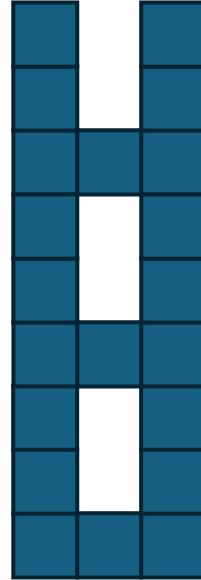
Stage 1



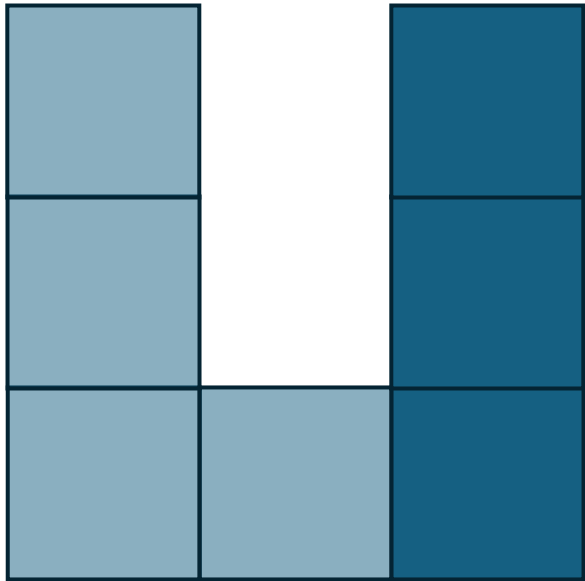
Discrete Self-Similar Fractal



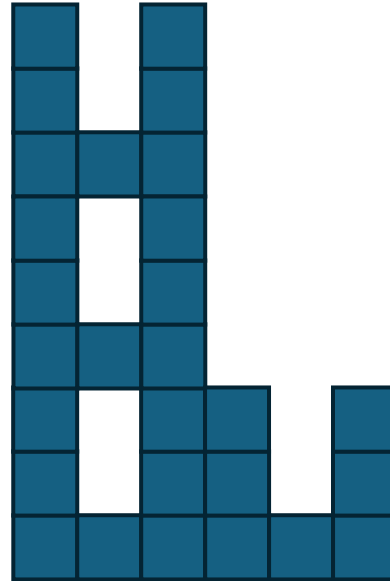
Stage 1



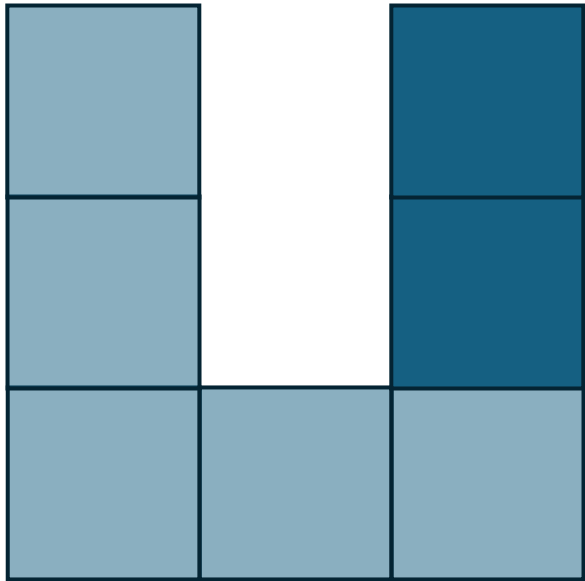
Discrete Self-Similar Fractal



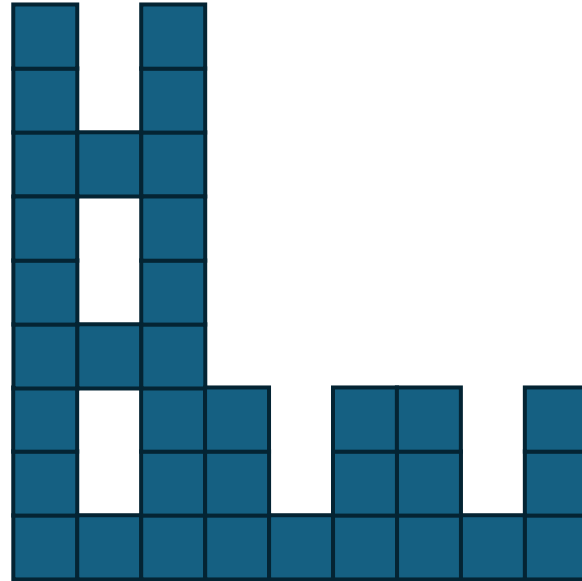
Stage 1



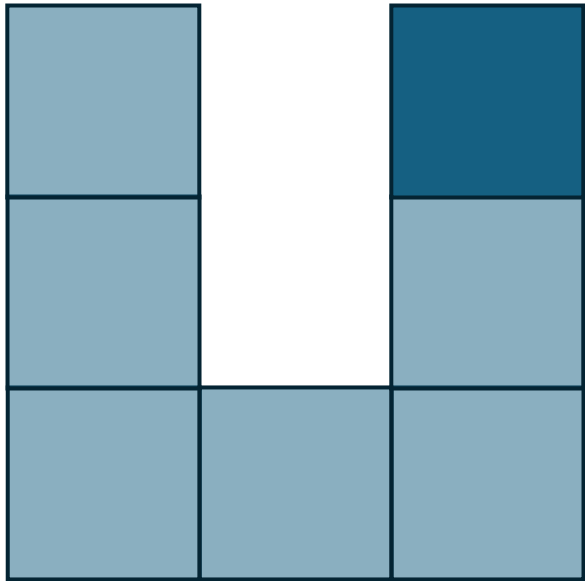
Discrete Self-Similar Fractal



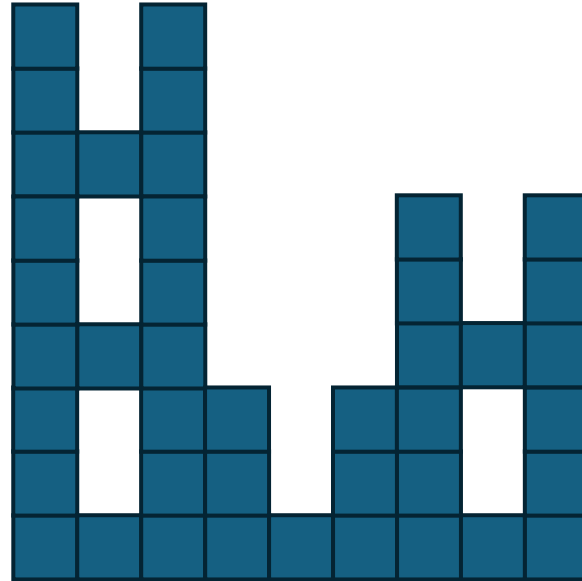
Stage 1



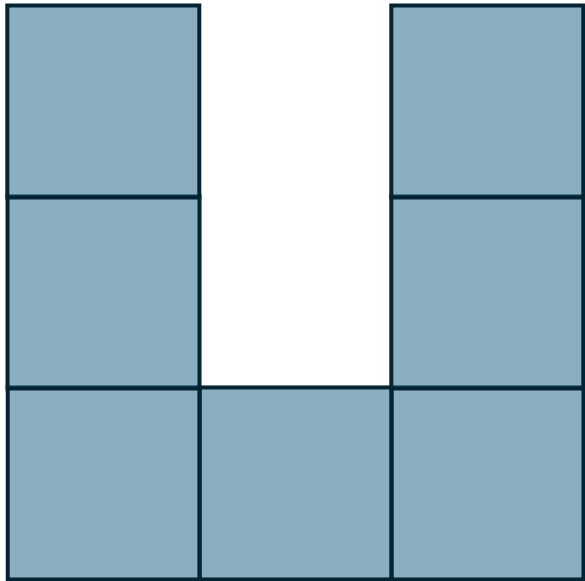
Discrete Self-Similar Fractal



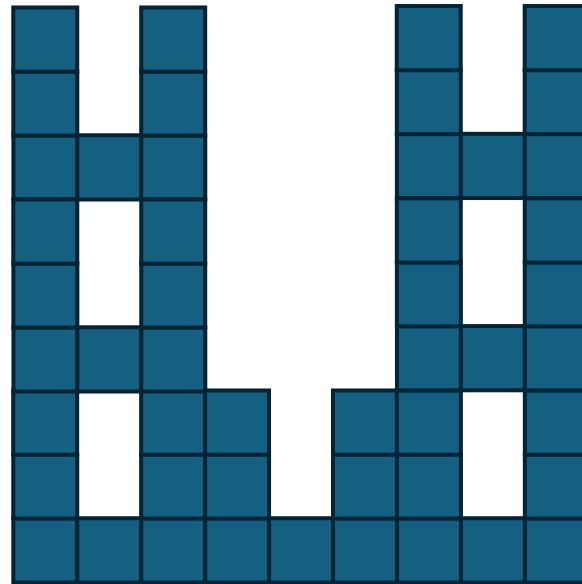
Stage 1



Discrete Self-Similar Fractal

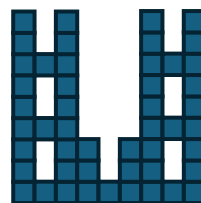
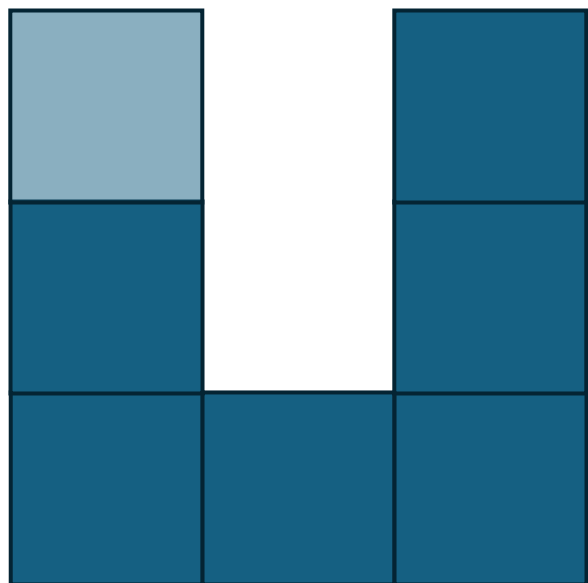


Stage 1

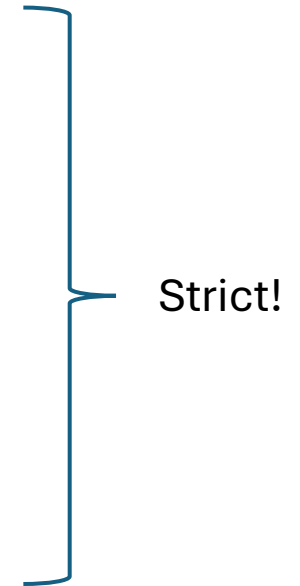
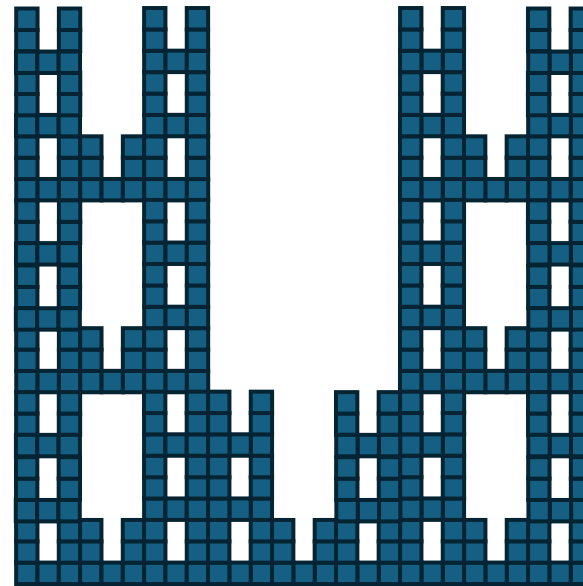
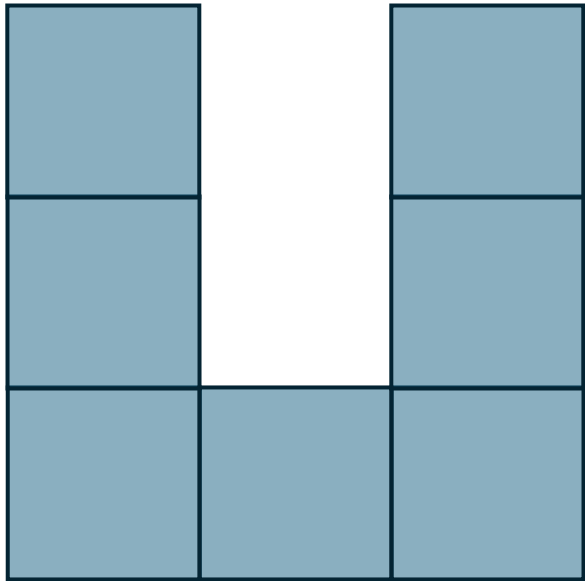


Stage 2

Discrete Self-Similar Fractal

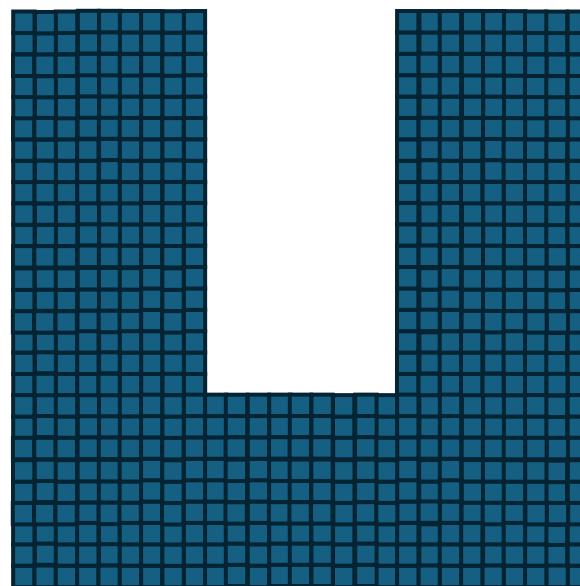
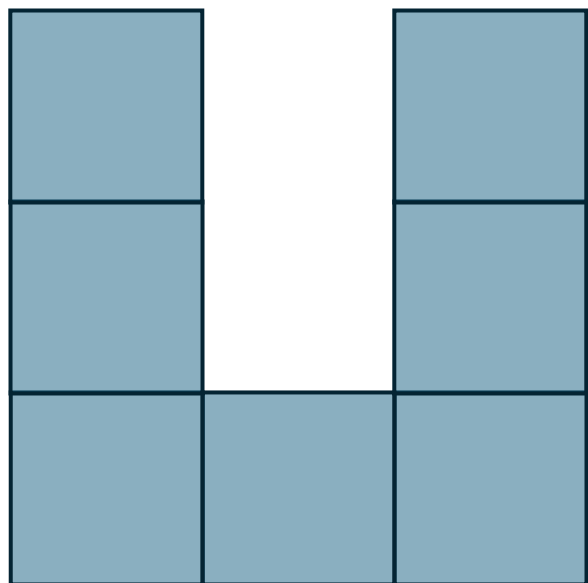


Discrete Self-Similar Fractal

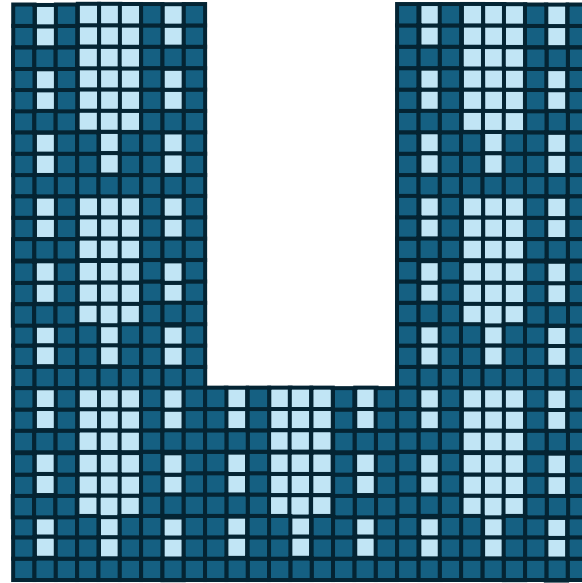
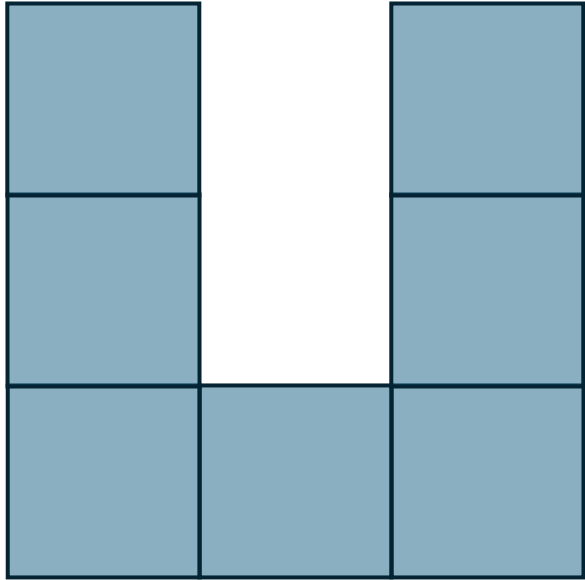


Stage 3

Discrete Self-Similar Fractal



Discrete Self-Similar Fractal



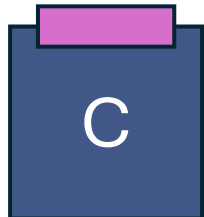
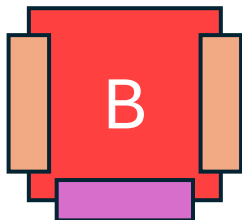
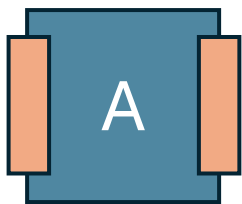
Weak!

Previous Work

Why is this important?

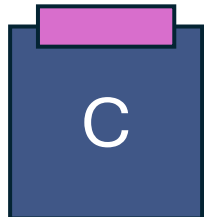
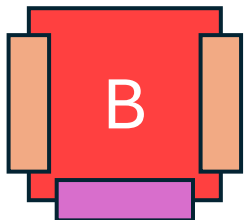
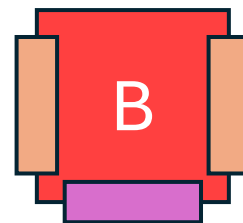
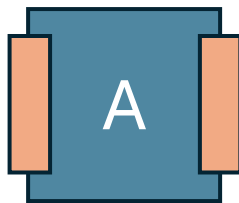
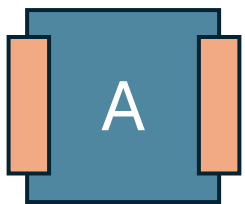
Tile Self-Assembly

Tiles:



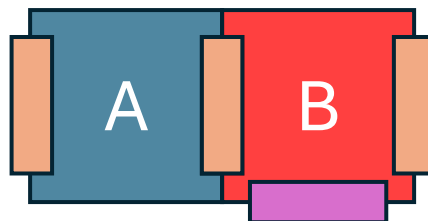
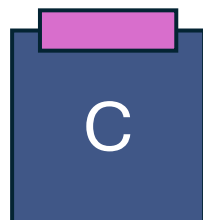
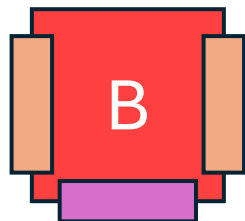
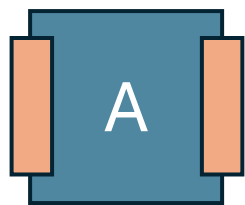
Tile Self-Assembly

Tiles:



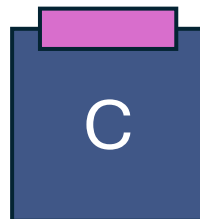
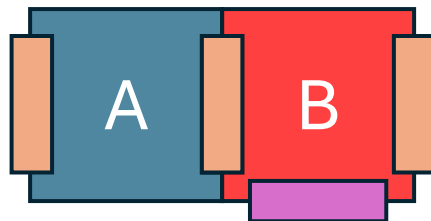
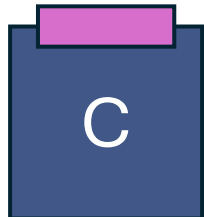
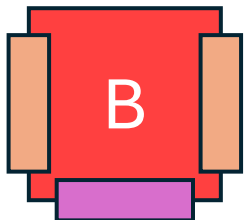
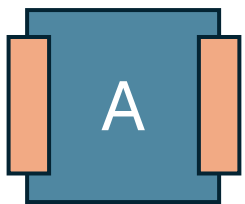
Tile Self-Assembly

Tiles:



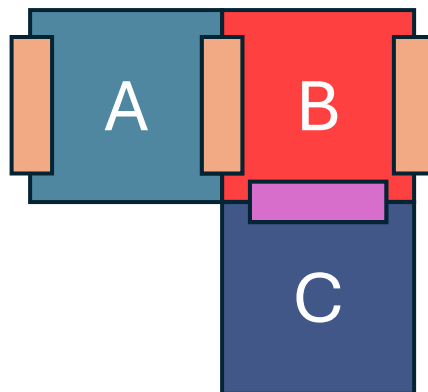
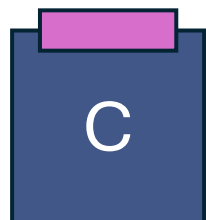
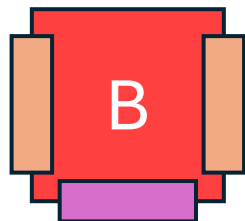
Tile Self-Assembly

Tiles:



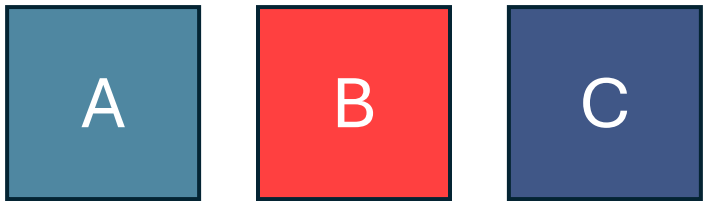
Tile Self-Assembly

Tiles:

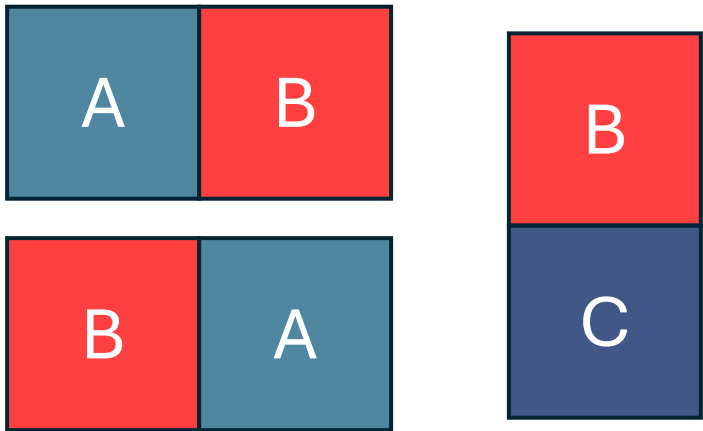


Tile Self-Assembly

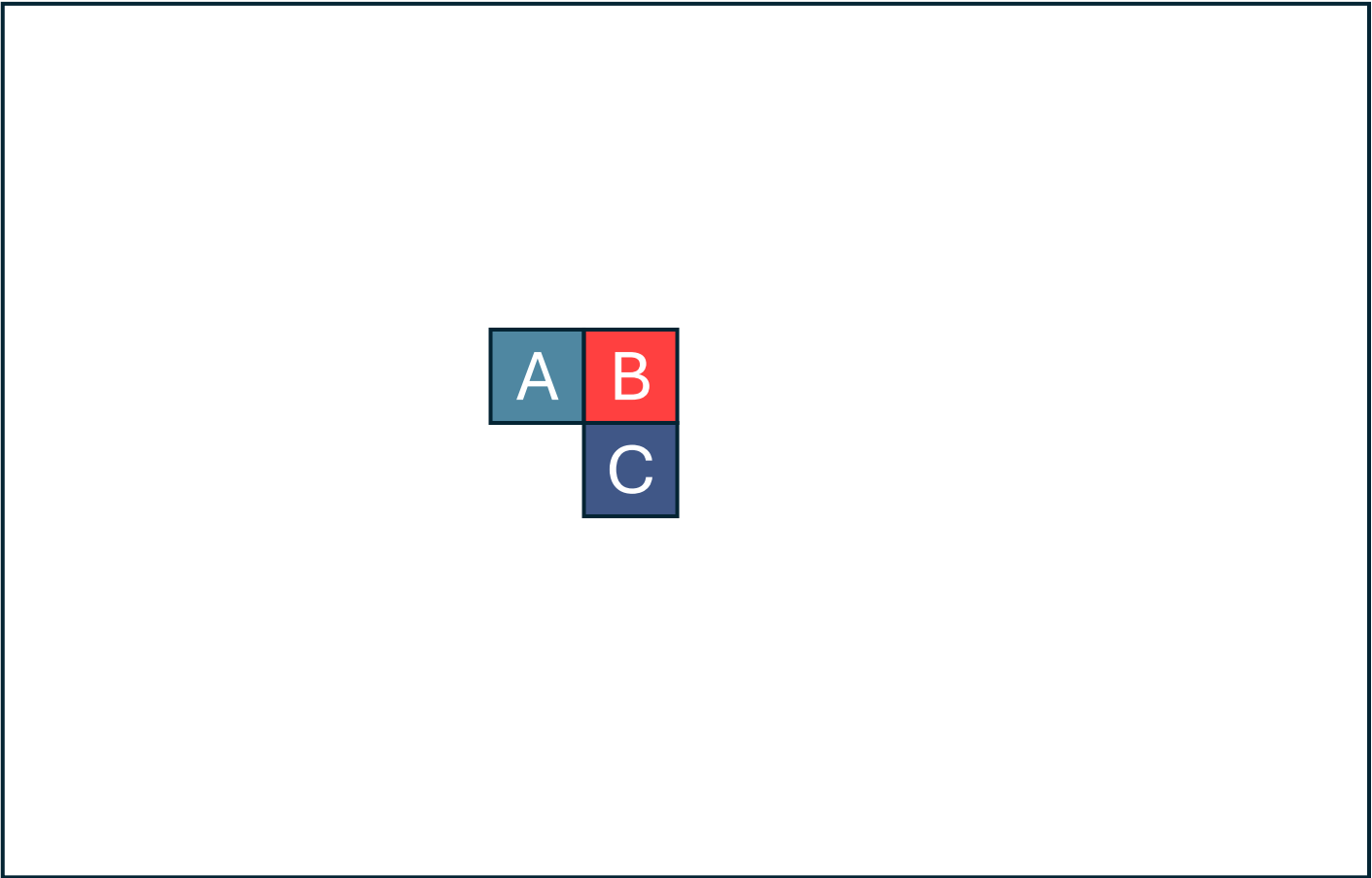
Tiles:



Affinities:



System:

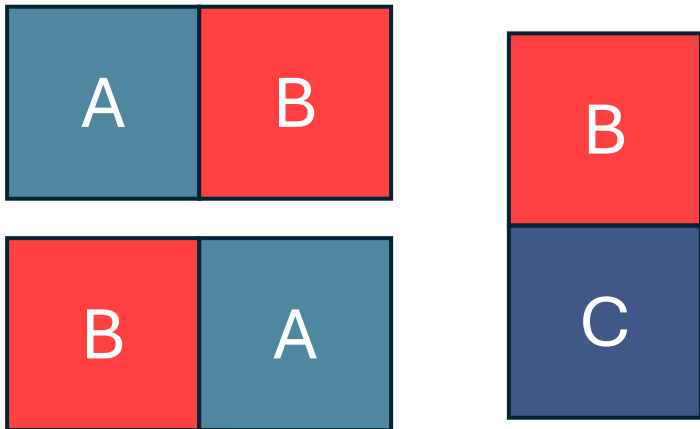


Tile Self-Assembly

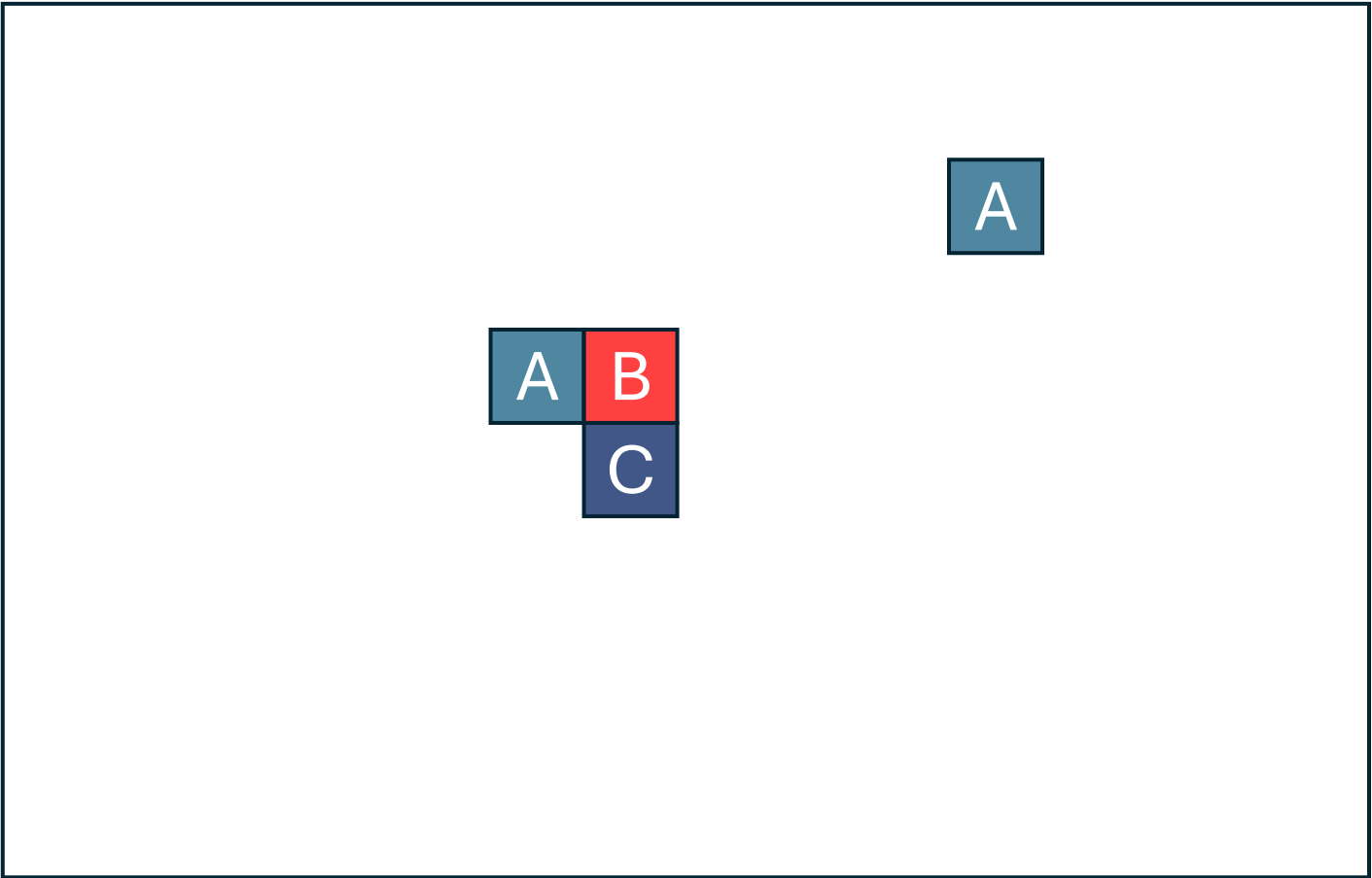
Tiles:



Affinities:

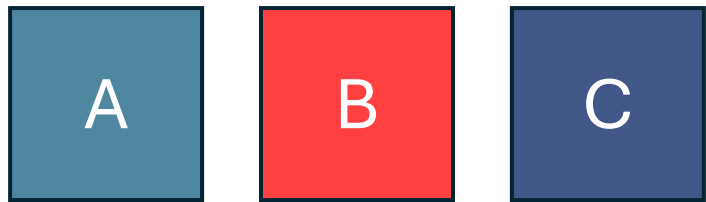


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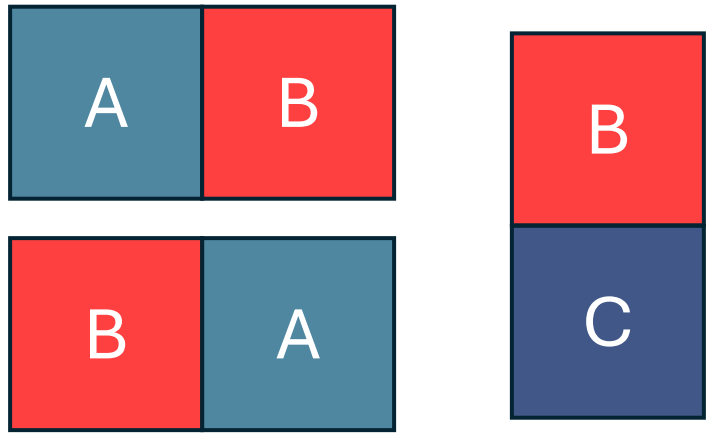


Tile Self-Assembly

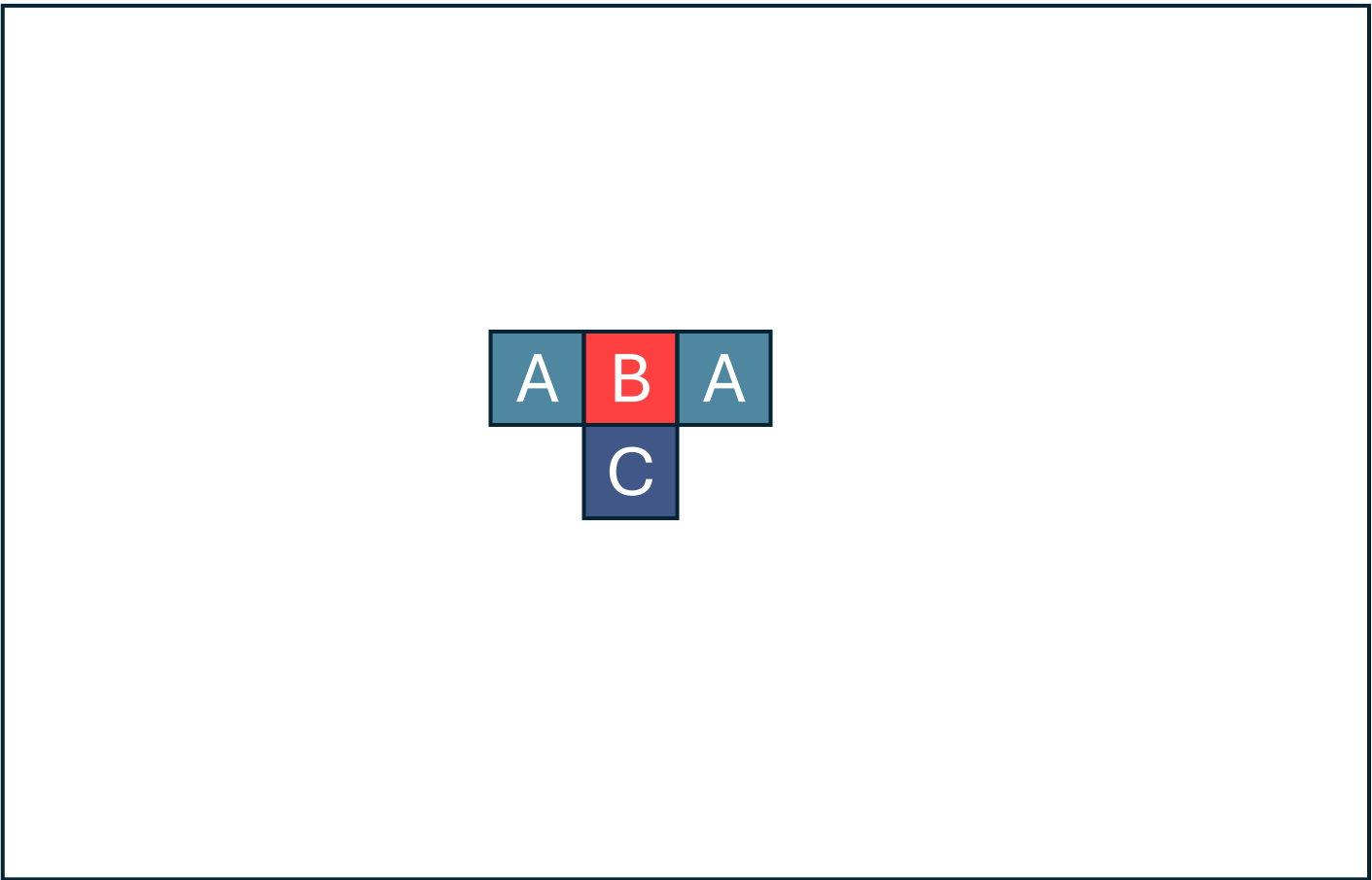
Tiles:



Affinities:



System:

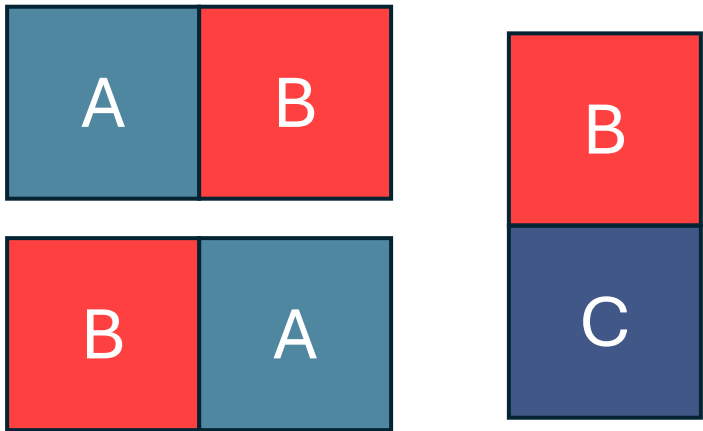


Tile Self-Assembly

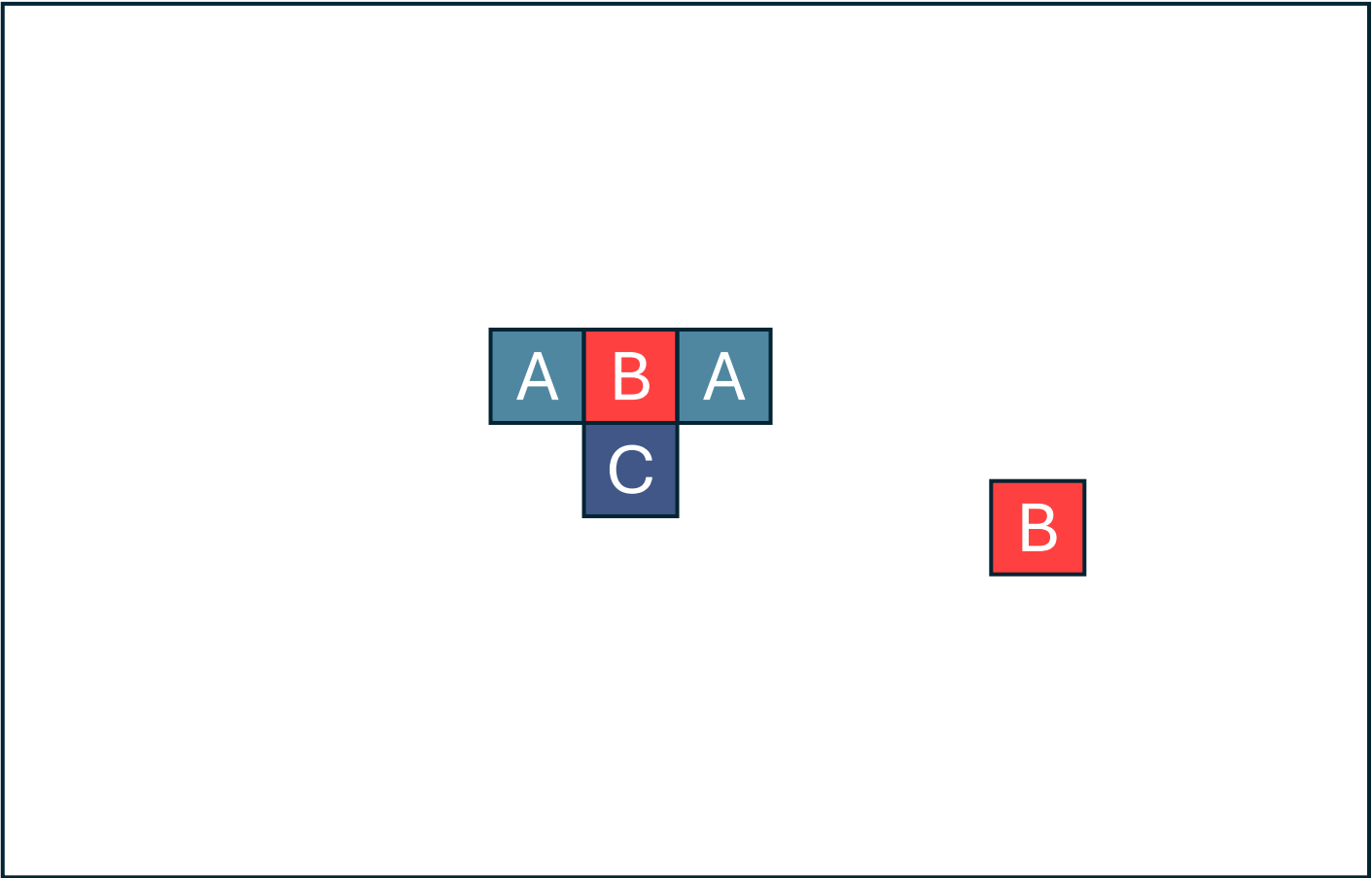
Tiles:



Affinities:



System:

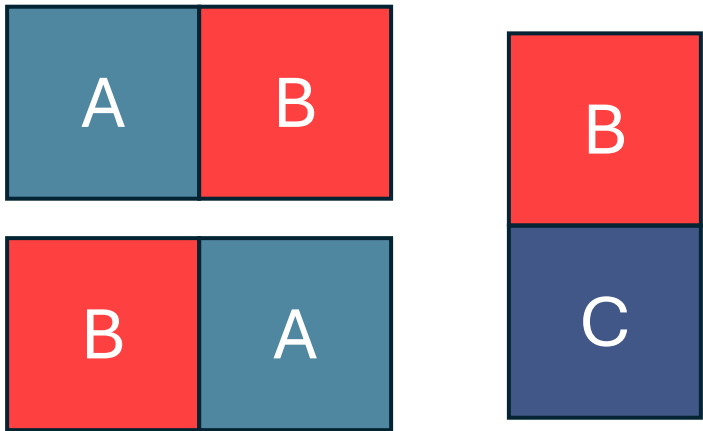


Tile Self-Assembly

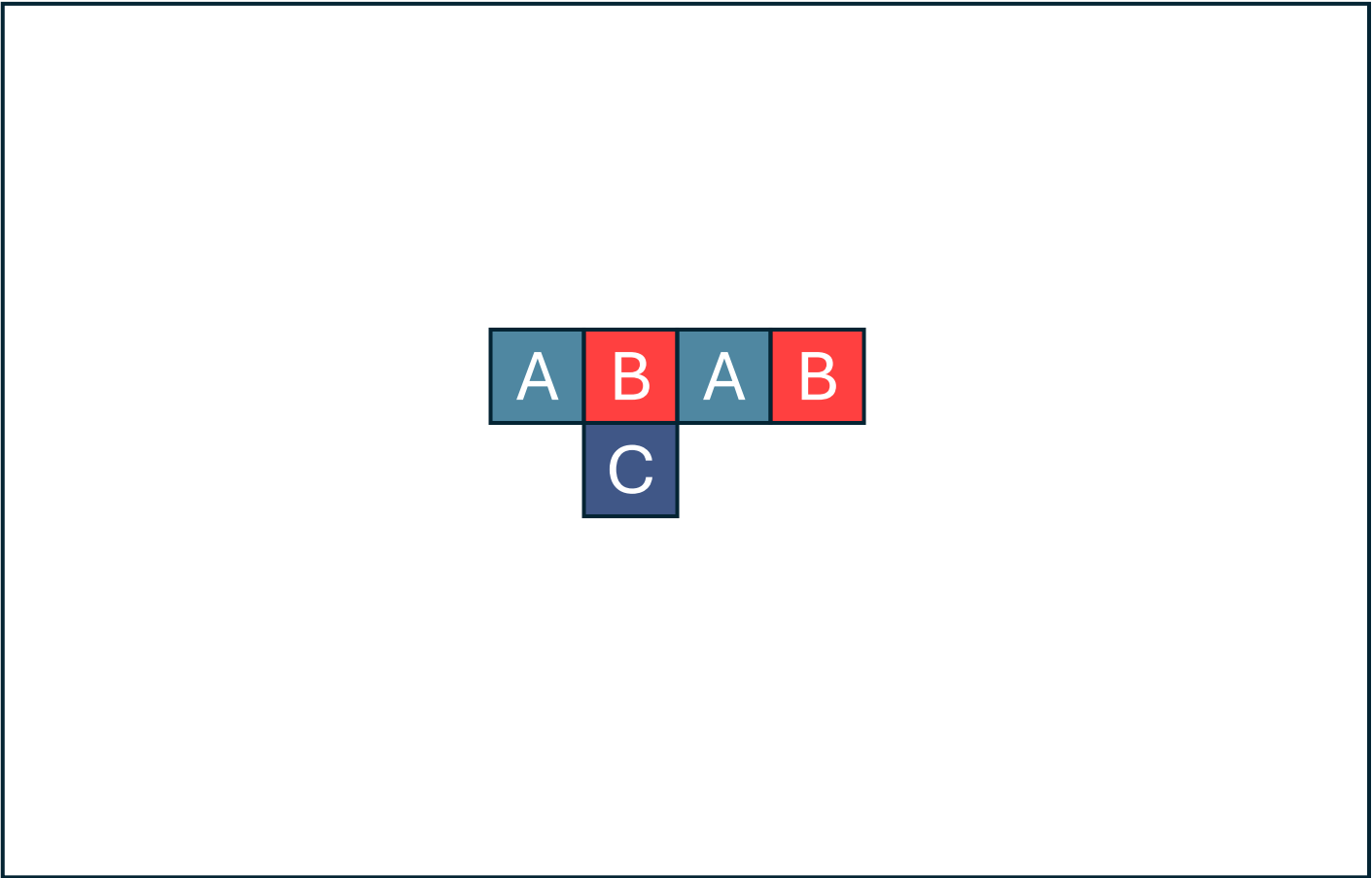
Tiles:



Affinities:

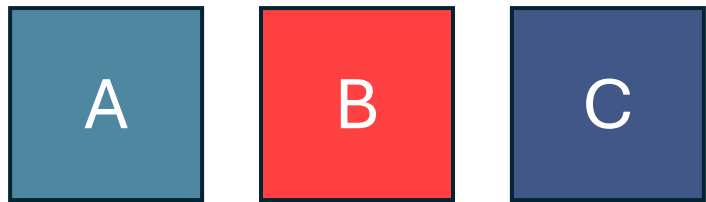


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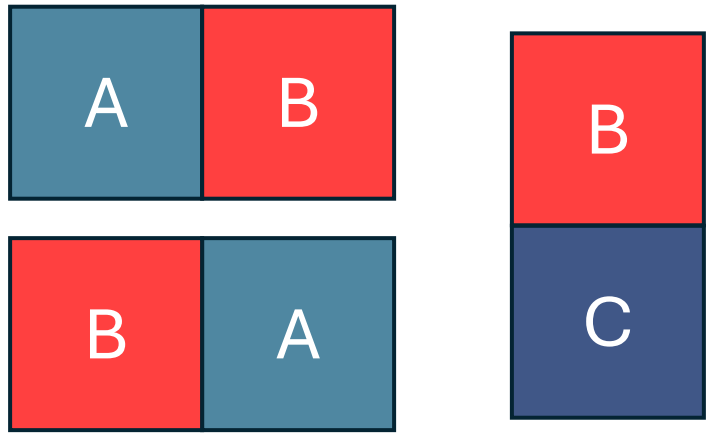


Tile Self-Assembly

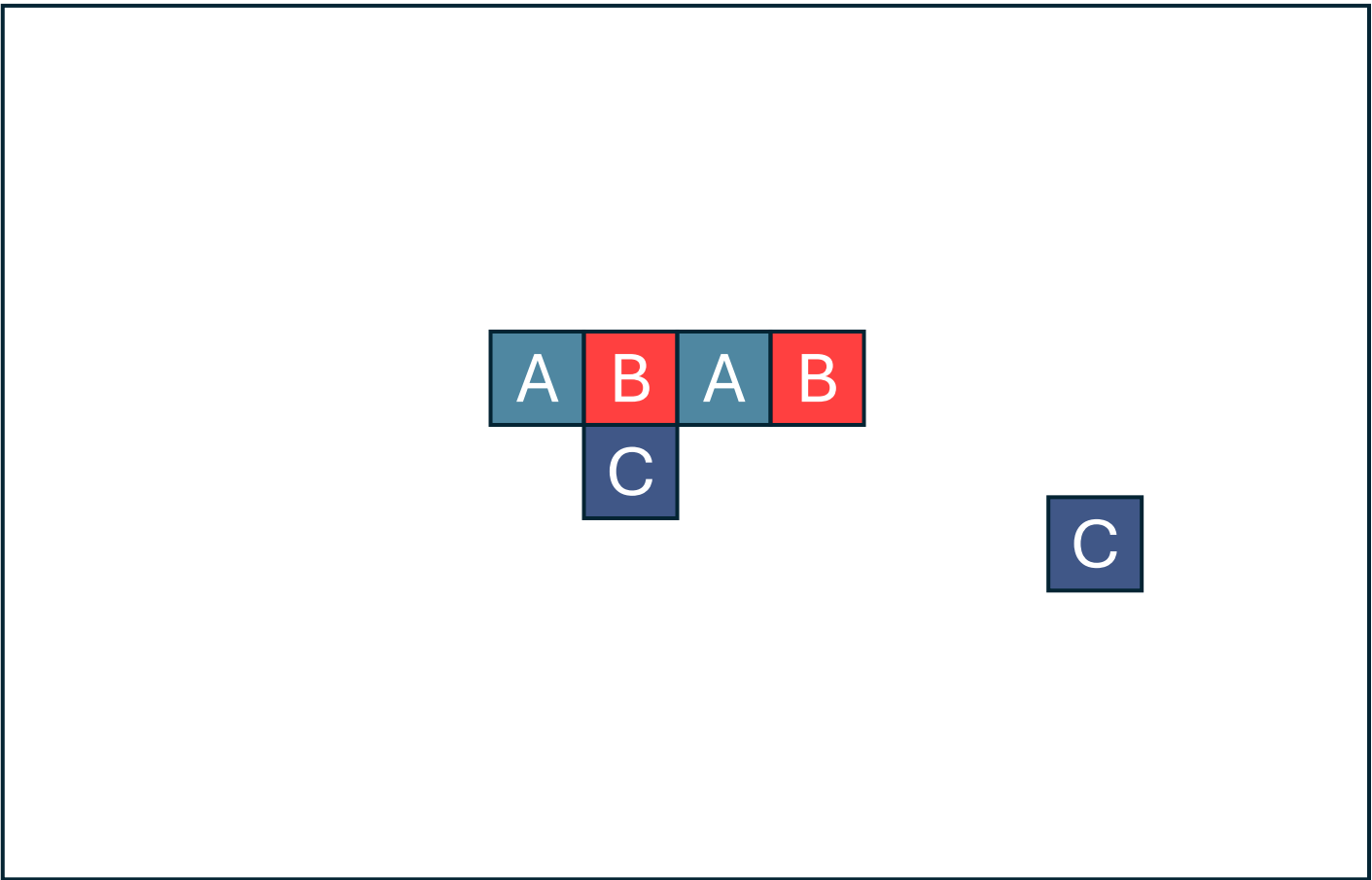
Tiles:



Affinities:

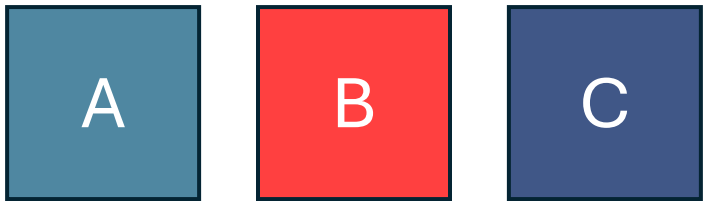


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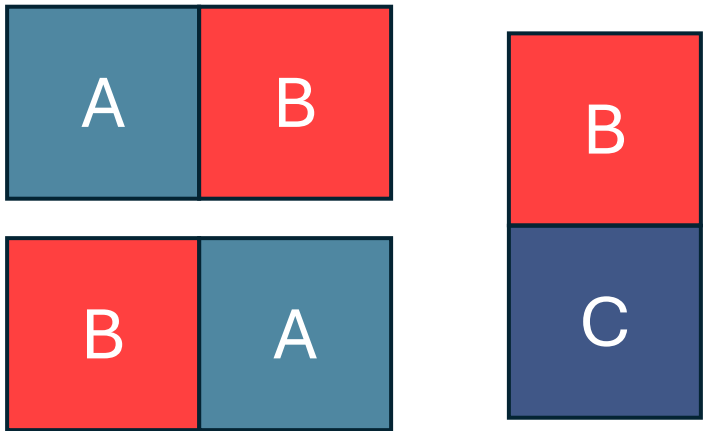


Tile Self-Assembly

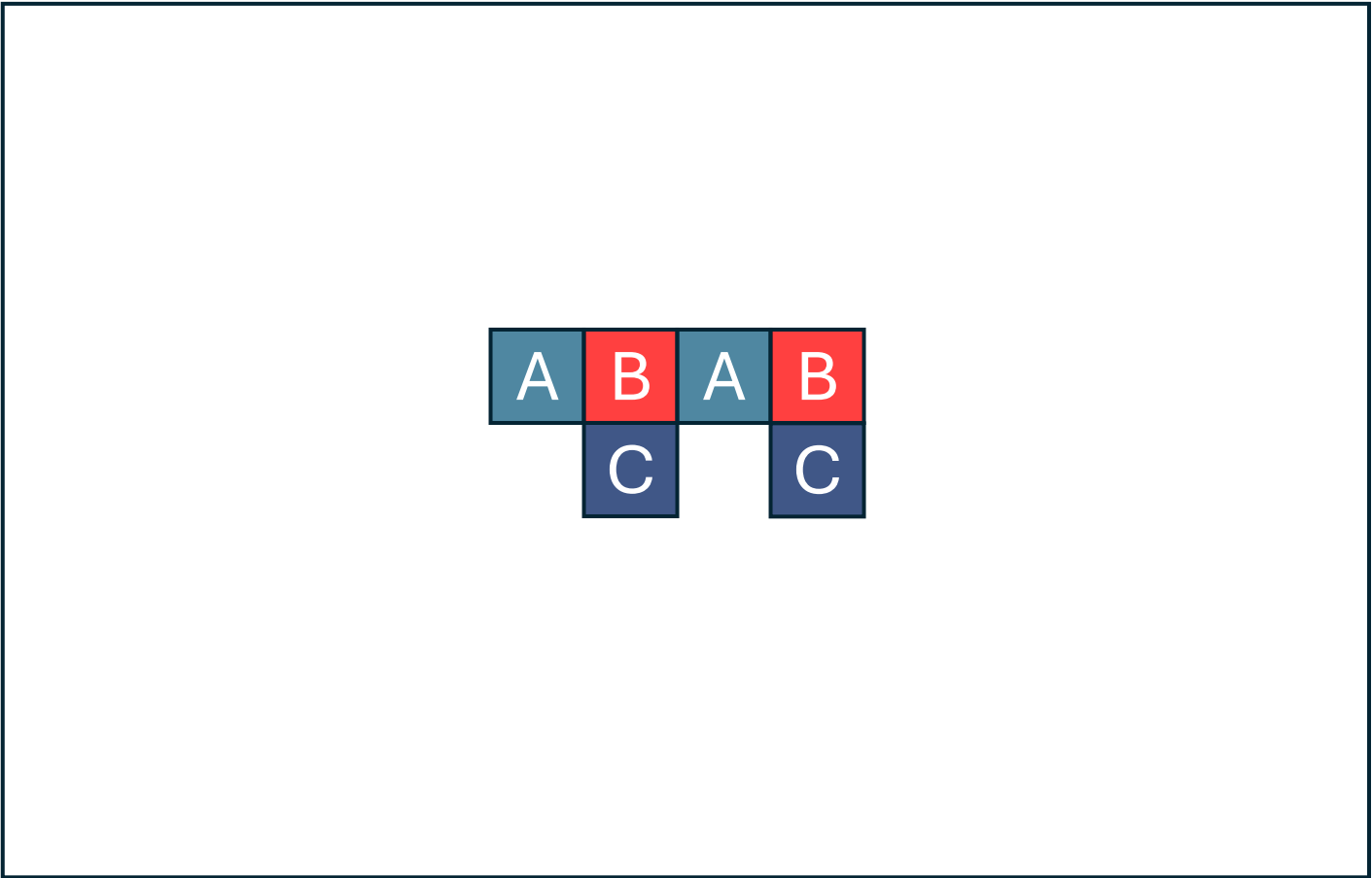
Tiles:



Affinities:



System:



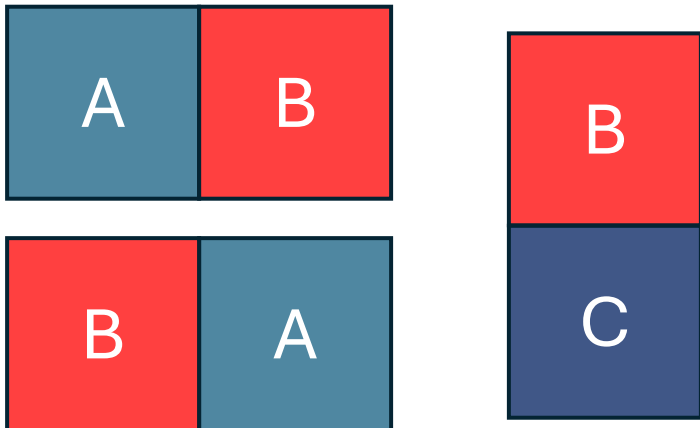
The aTAM (abstract Tile Assembly Model)

Tiles:

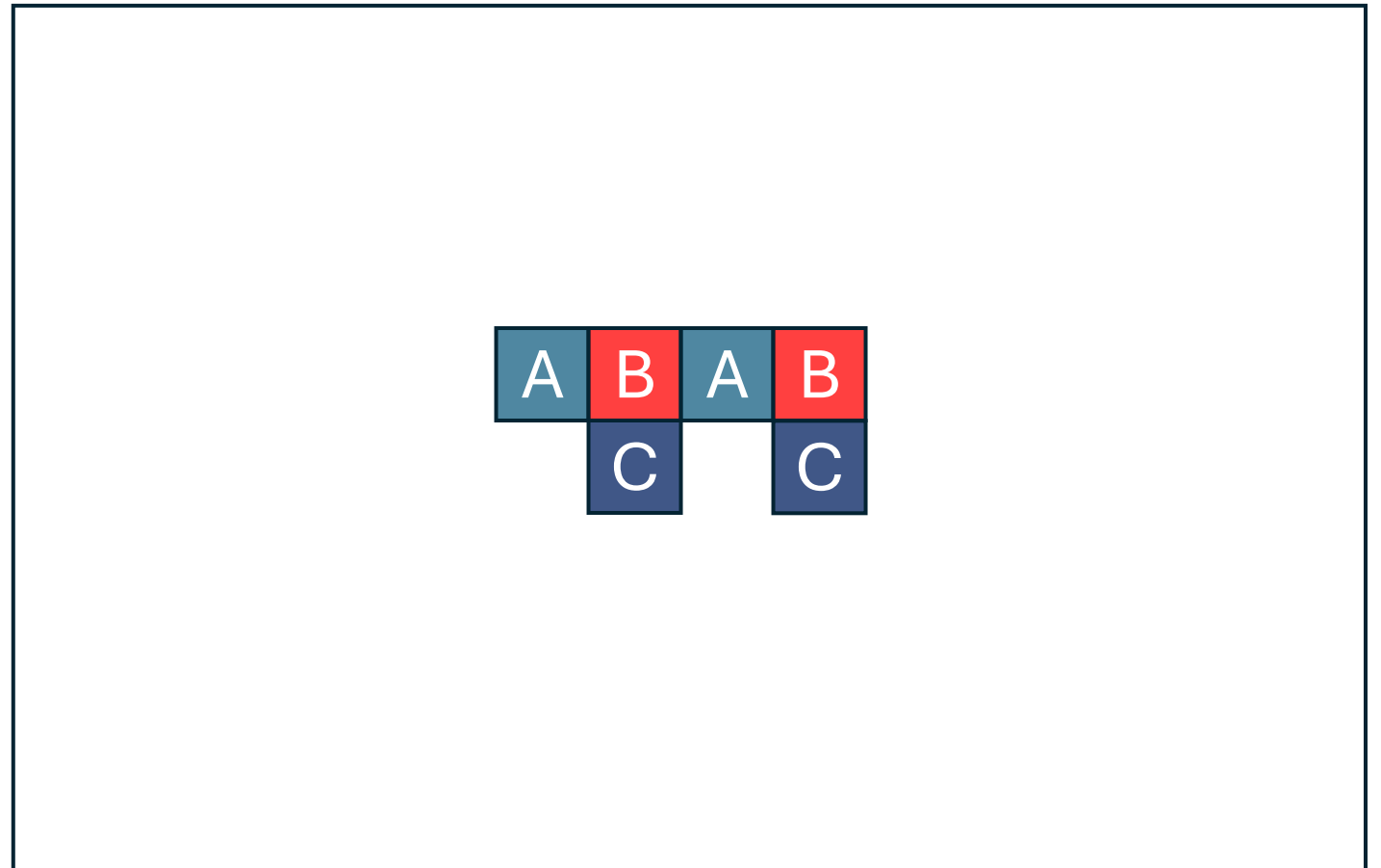


Affinities:

Temperature: τ

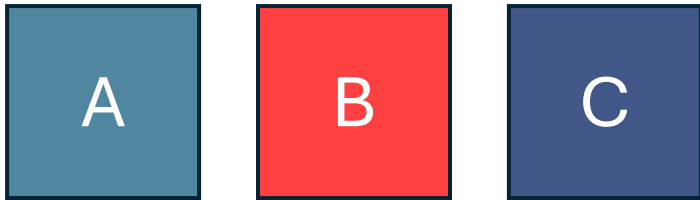


System:



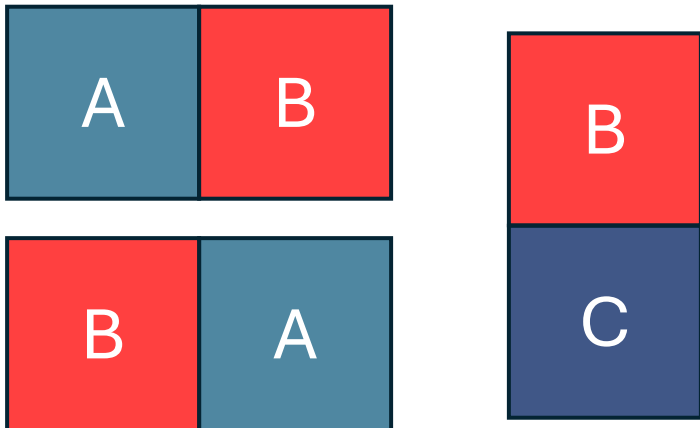
The aTAM (abstract Tile Assembly Model)

Tiles:

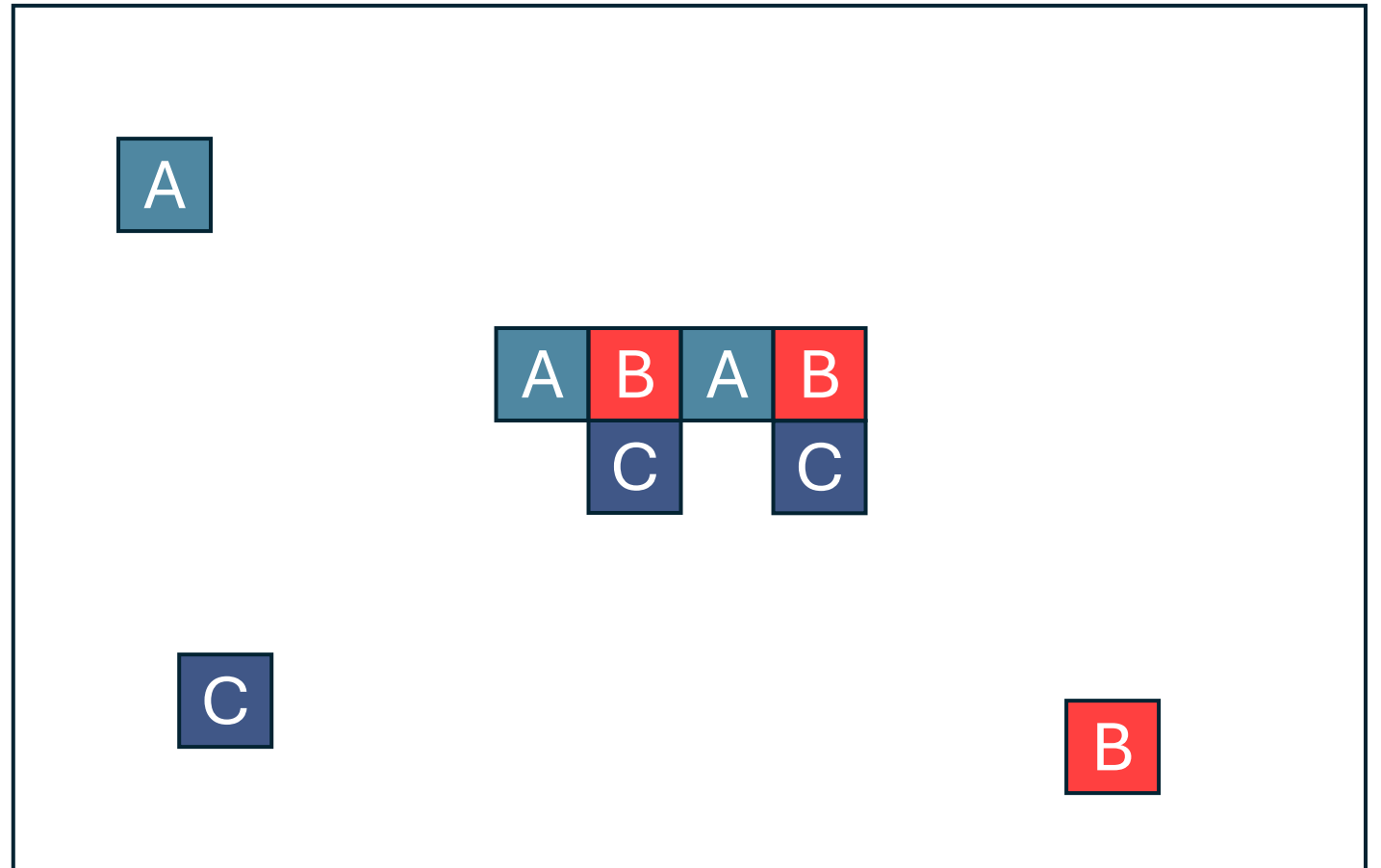


Affinities:

Temperature: τ



System:



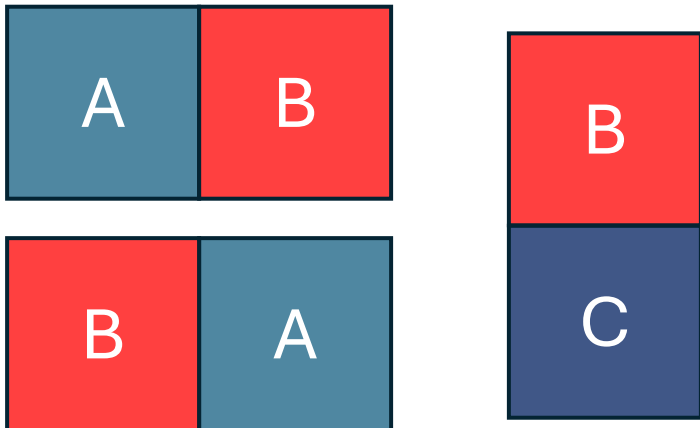
The aTAM (abstract Tile Assembly Model)

Tiles:

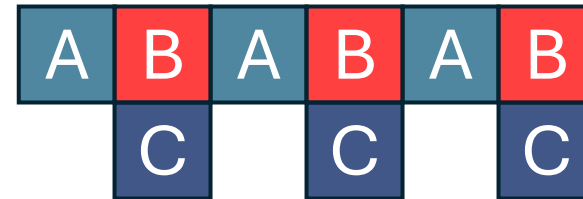


Affinities:

Temperature: τ



System:



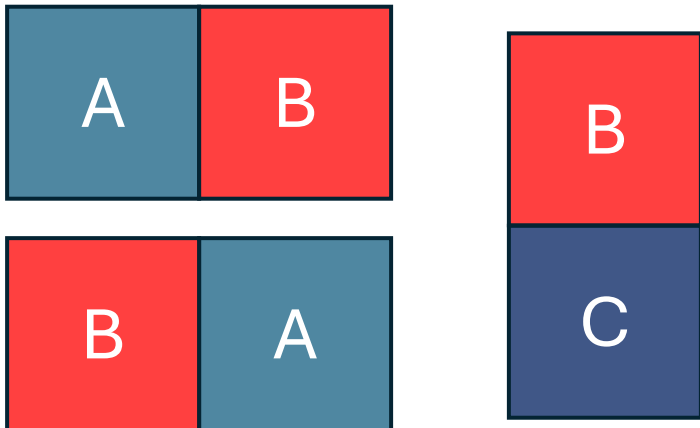
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Tiles:

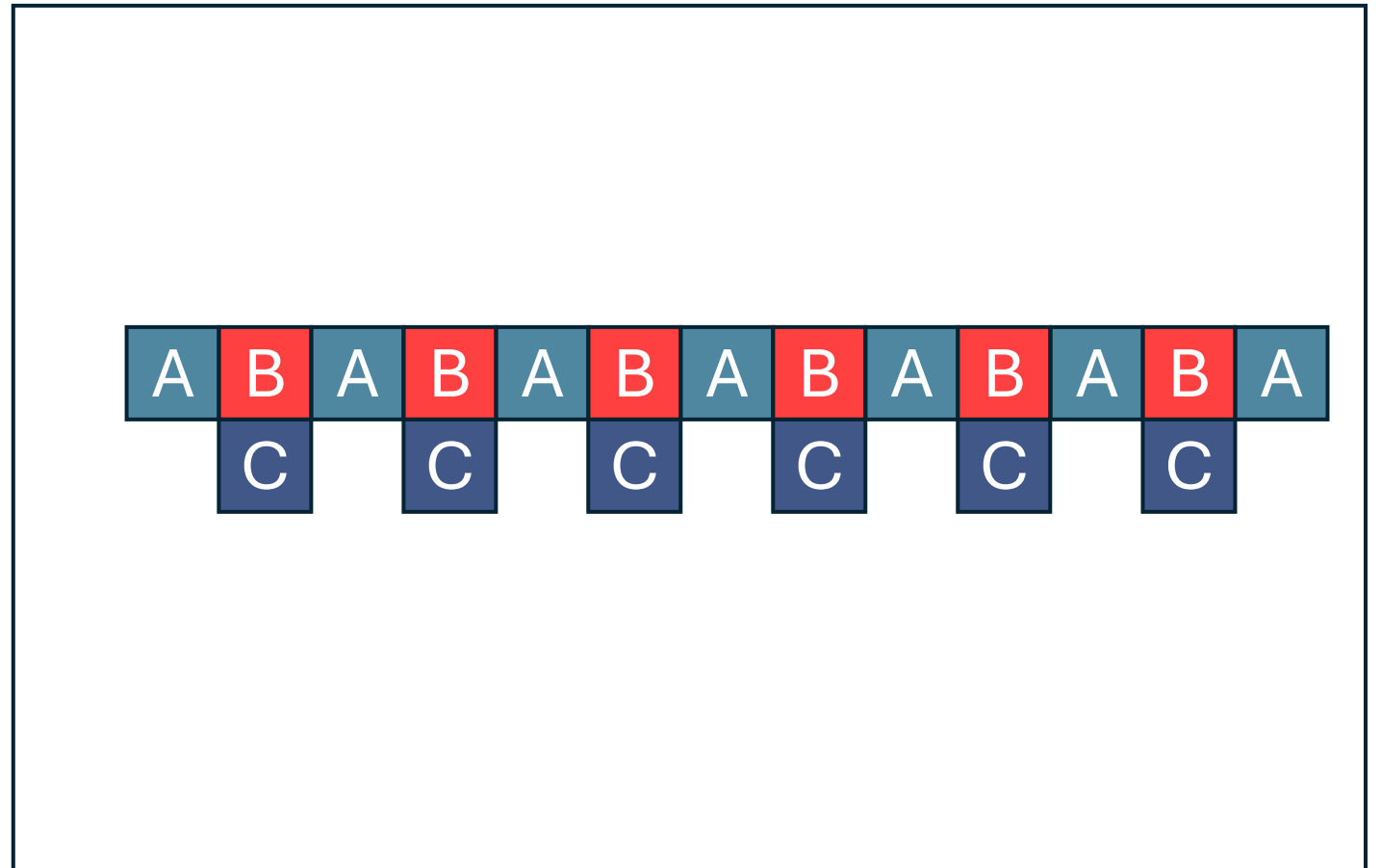


Affinities:

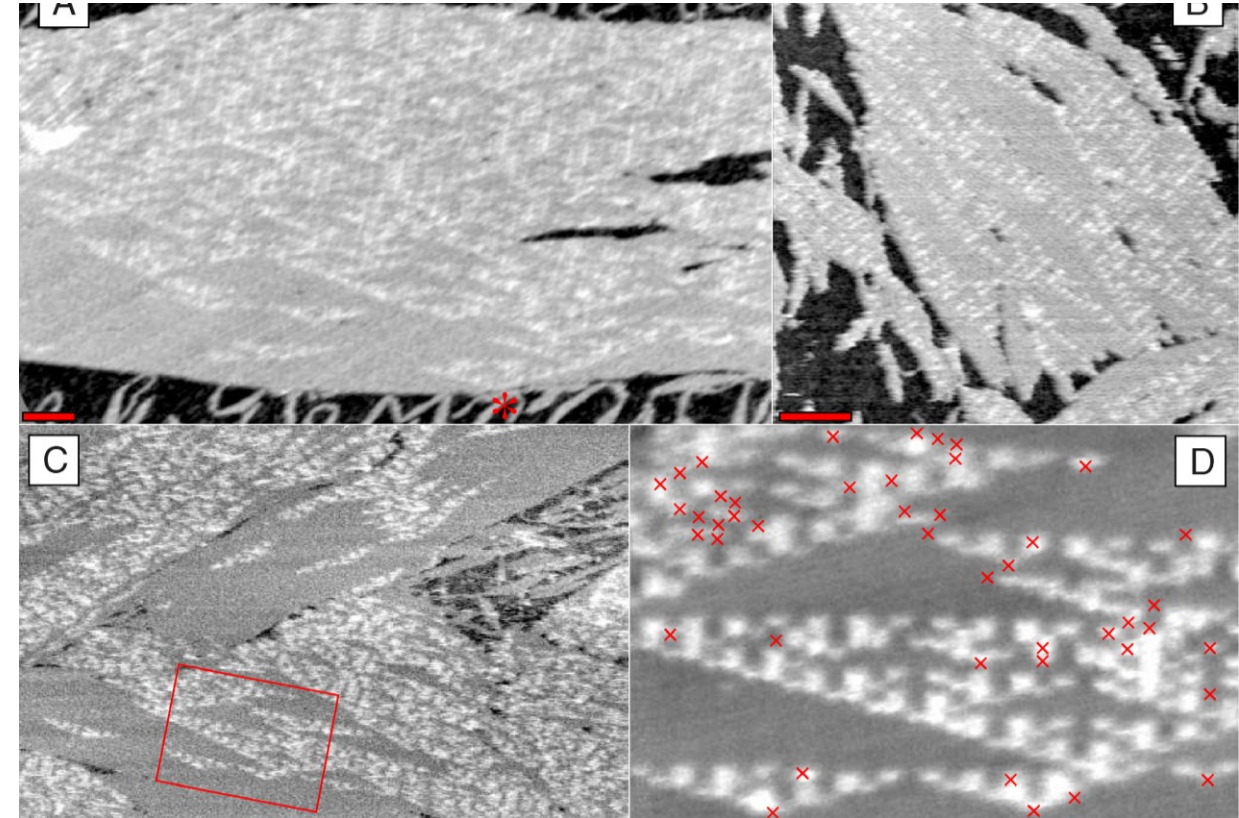
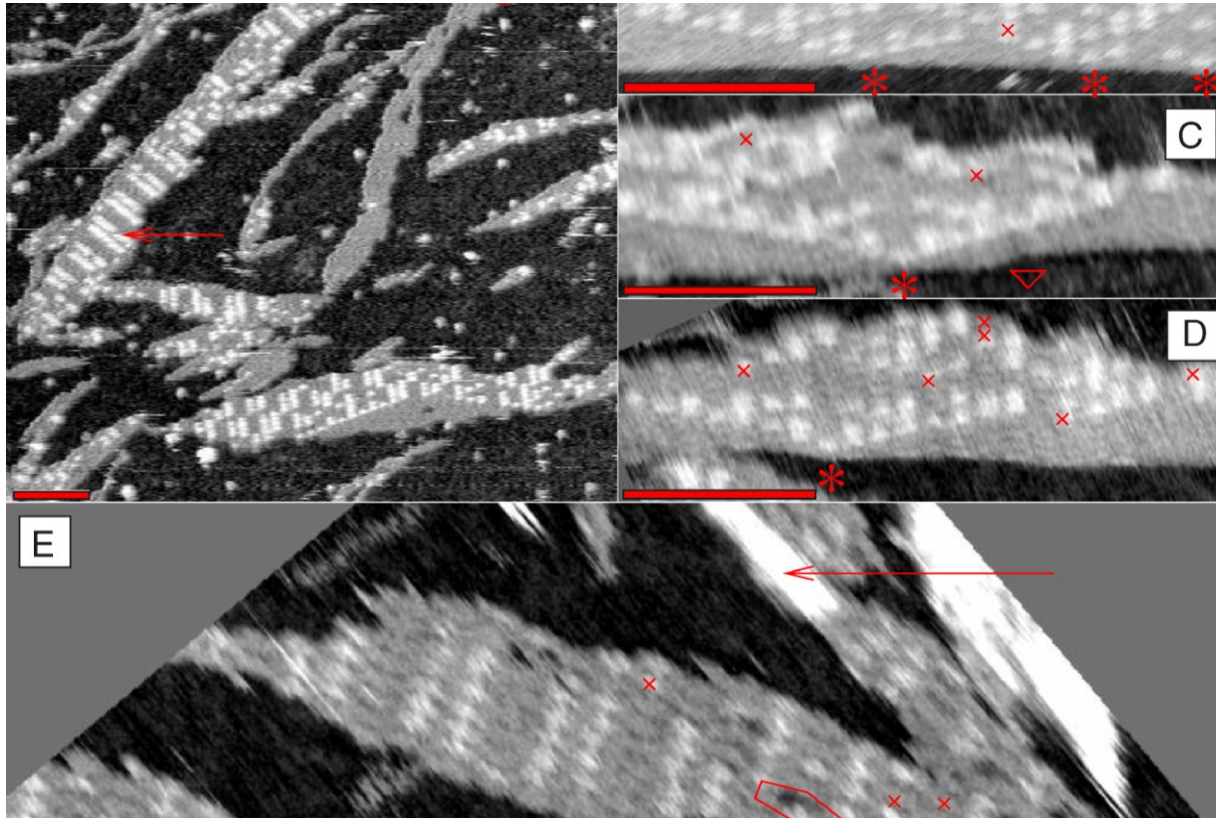
Temperature: τ



System:



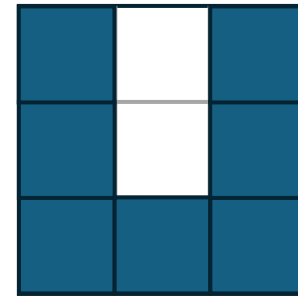
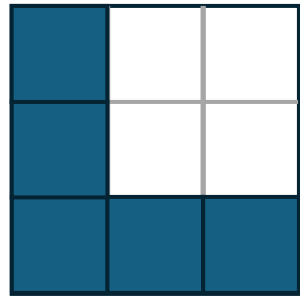
What is known (aTAM)



Rothemund PWK, Papadakis N, Winfree E (2004) Algorithmic Self-Assembly of DNA Sierpinski Triangles. PLoS Biol 2(12): e424.

What is known (aTAM)

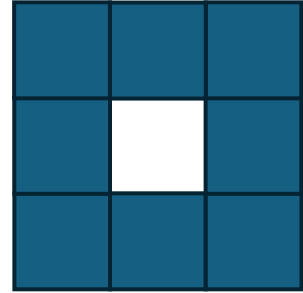
- **2010** [M. J. Patitz, S. M. Summers. Self-assembly of discrete self-similar fractals]:
 1. Any “L” shape does not strictly self assemble in the aTAM (any temp)
 2. No fractal weakly self assembles at temperature 1
 3. “Nice” fractals can be weakly self assembled at higher temps



- **2025** [F. Becker, D. Hader, M. J. Patitz. Strict self-assembly of discrete self-similar fractals in the abstract tile-assembly model]:
 1. A full characterization of which DSSFs are buildable in the aTAM.

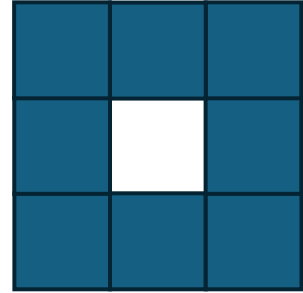
Variations of the aTAM

- 2HAM (2-Handed Assembly Model):
 1. 2 different assemblies can attach
 2. Can finitely assemble a larger class of shapes
- STAM (Signal-Passing Tile Assembly Model):
 1. Glues can turn “on” and “off”
 2. Detachment is allowed



Variations of the aTAM

- 2HAM (2-Handed Assembly Model):
 1. 2 different assemblies can attach
 2. Can finitely assemble a larger class of shapes
- STAM (Signal-Passing Tile Assembly Model):
 1. Glues can turn “on” and “off”
 2. Detachment is allowed
 3. Any arbitrary fractal is buildable
 4. Without detachments, some shapes impossible



A Comparison

	aTAM	2HAM	STAM
Description	<ul style="list-style-type: none">- Single tile attachments- No detachments- Fixed glues	<ul style="list-style-type: none">- Up to 2 assembly attachments- No detachments- Fixed glues	<ul style="list-style-type: none">- Up to 2 assembly attachments- Detachments- Glues turn on/off
Results	<ul style="list-style-type: none">- No fractals weakly buildable (temp 1)- Some fractals not strictly buildable (temp > 1)	<ul style="list-style-type: none">- Finitely assemble larger class of shapes- Still some non-buildable fractals	<ul style="list-style-type: none">- All fractals strictly buildable- Without detachment: some impossible fractals

Question: What if adjacent tiles can infinitely change states?

Seeded Tile Automata

What is the model?

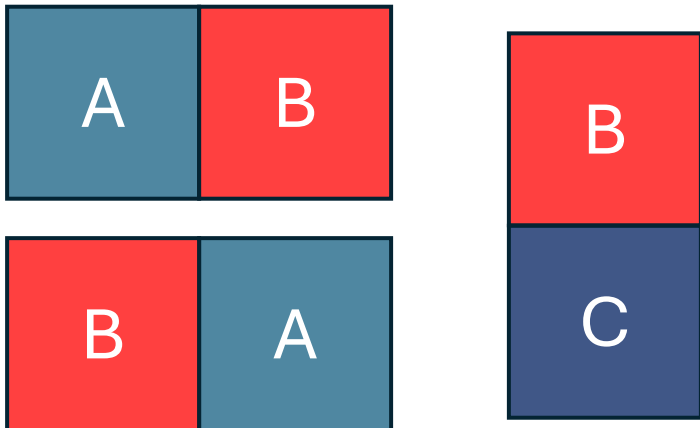
Seeded TA

Tiles:

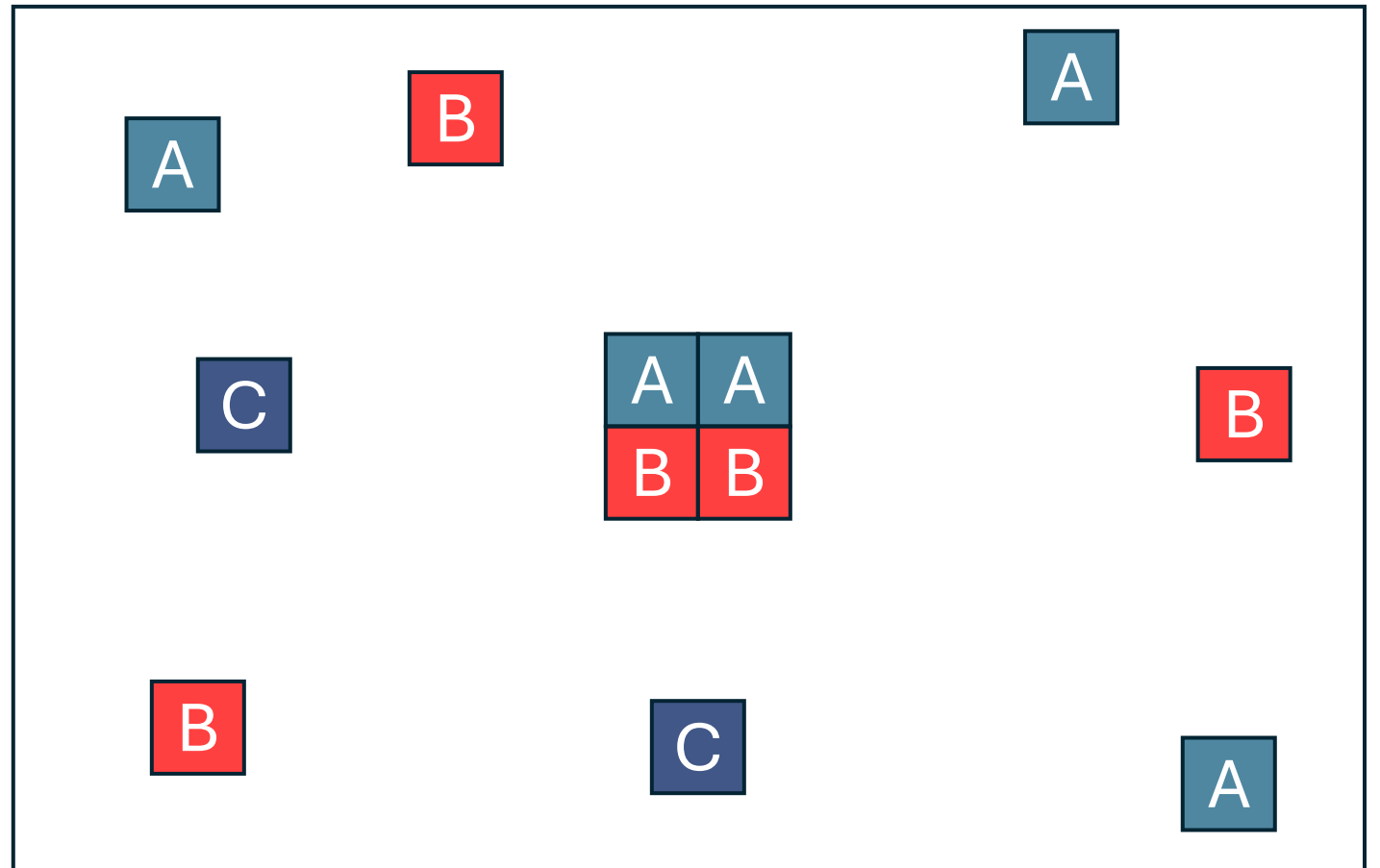


Affinities:

Temperature: τ



System:



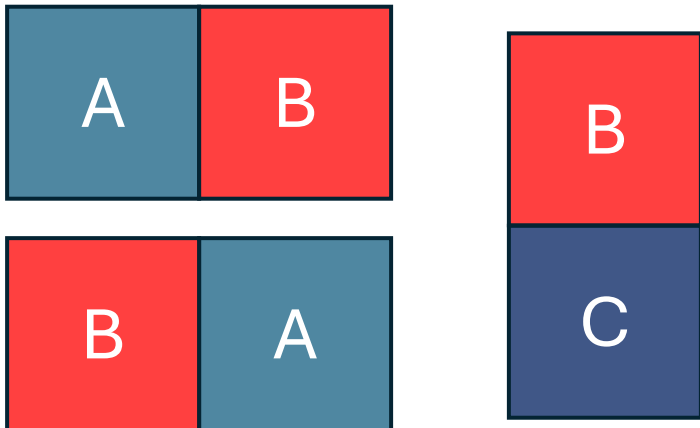
Seeded TA

Tiles:

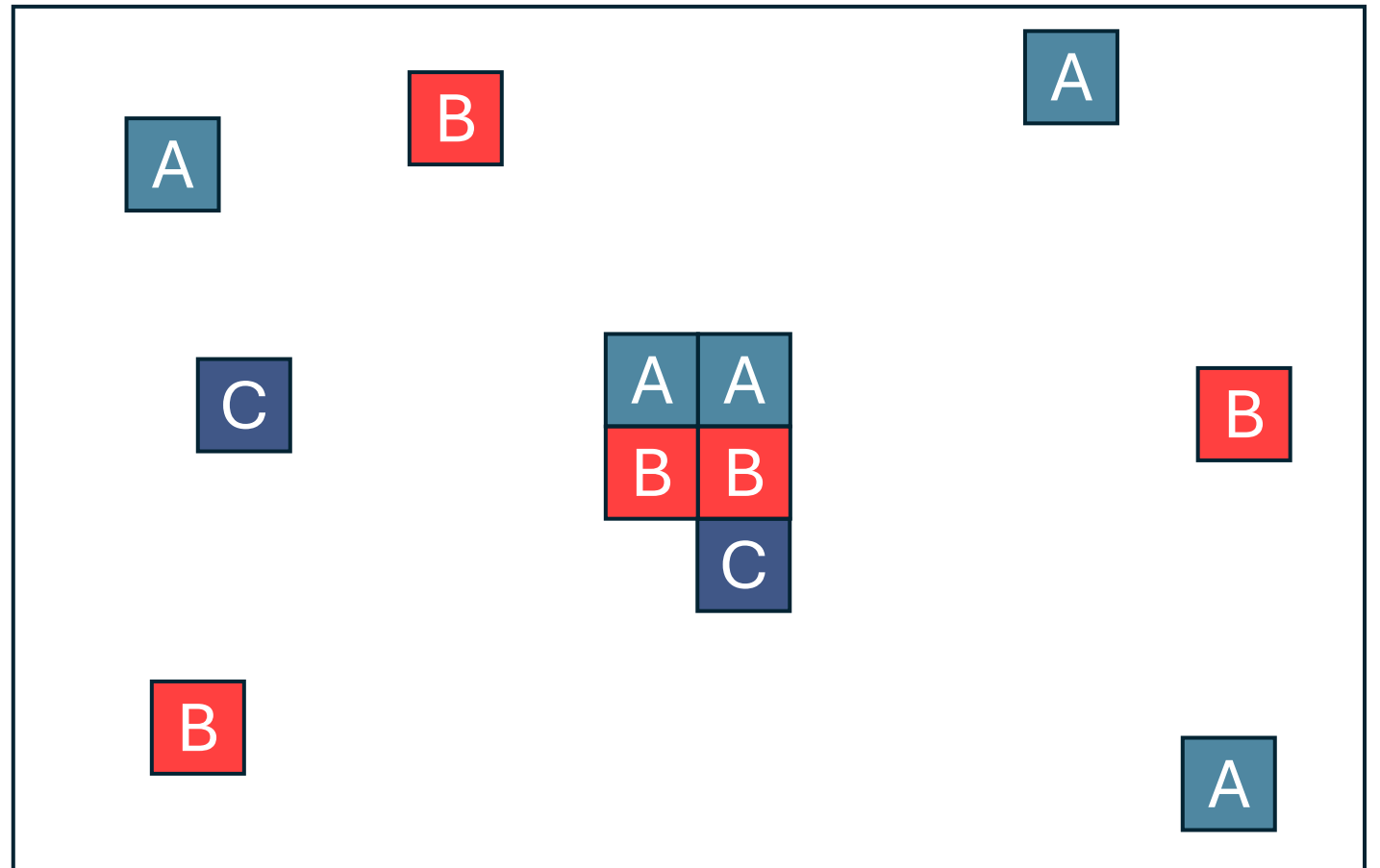


Affinities:

Temperature: τ

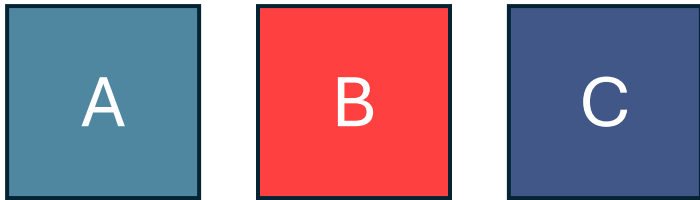


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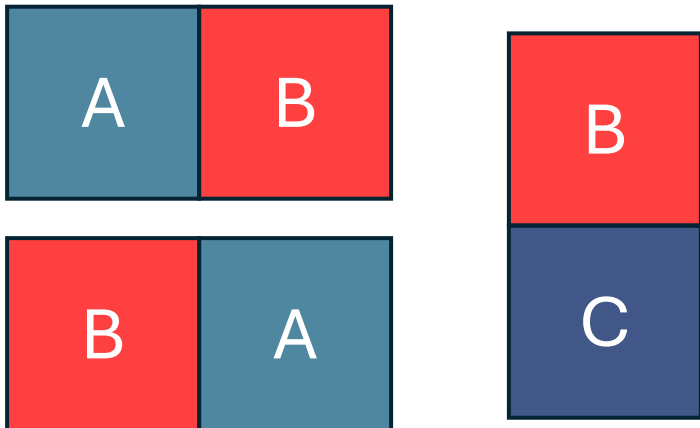
Seeded TA

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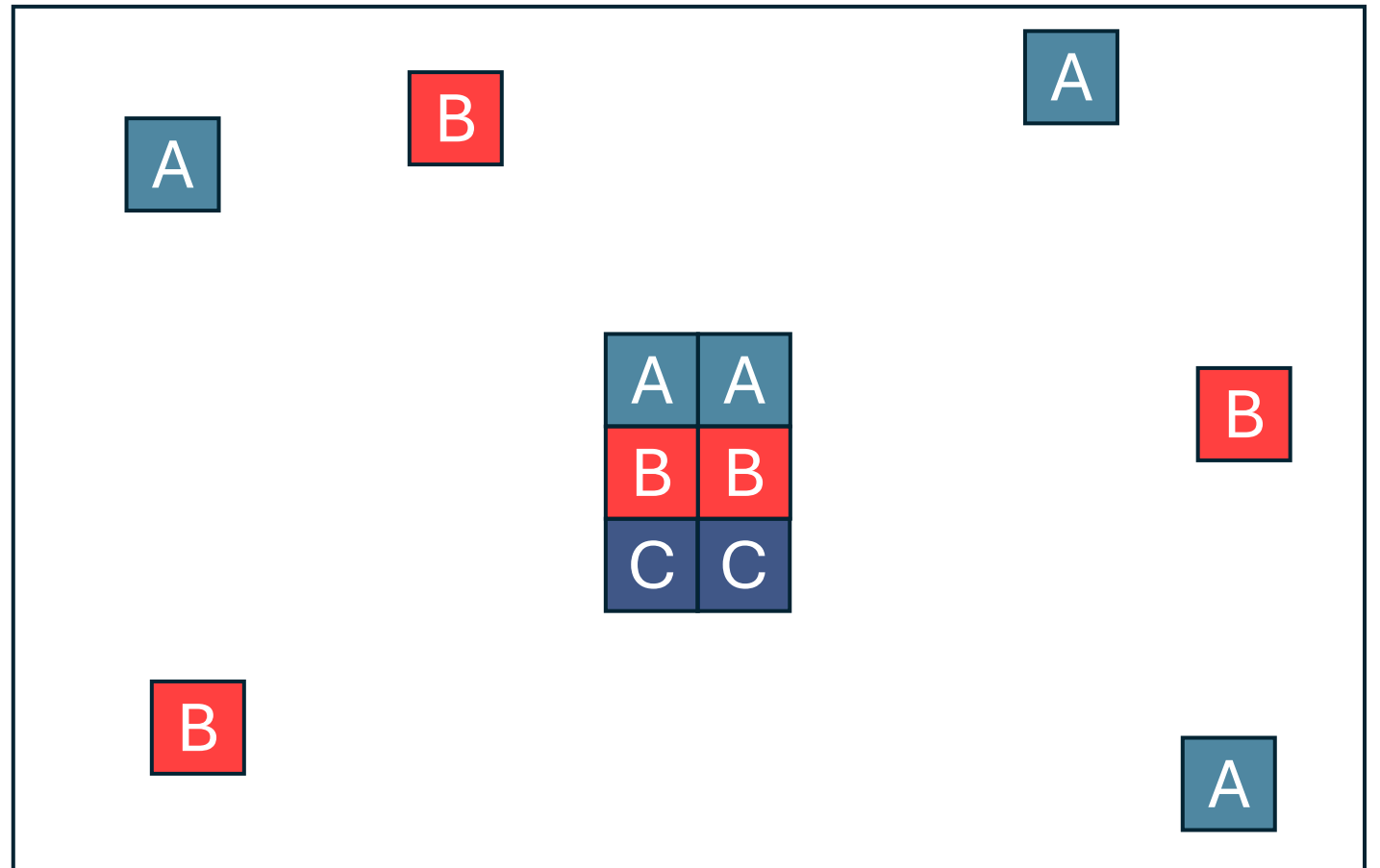


Affinities:

Temperature: τ

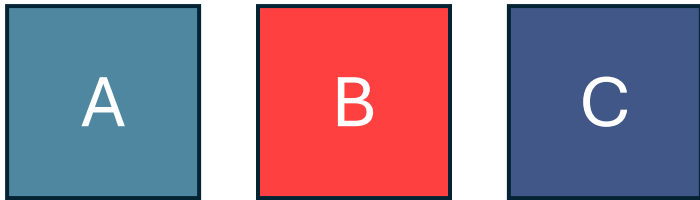


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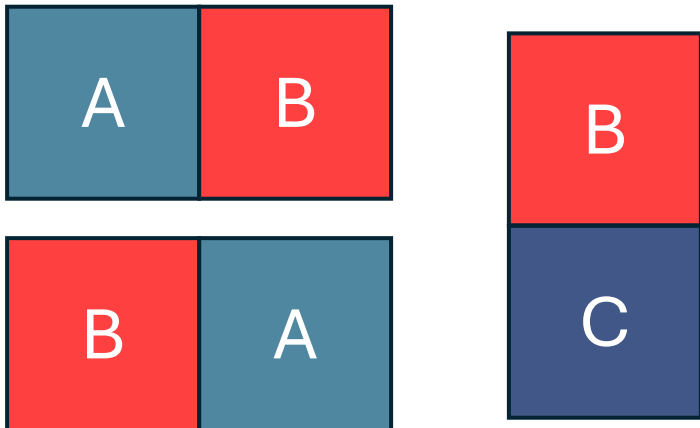
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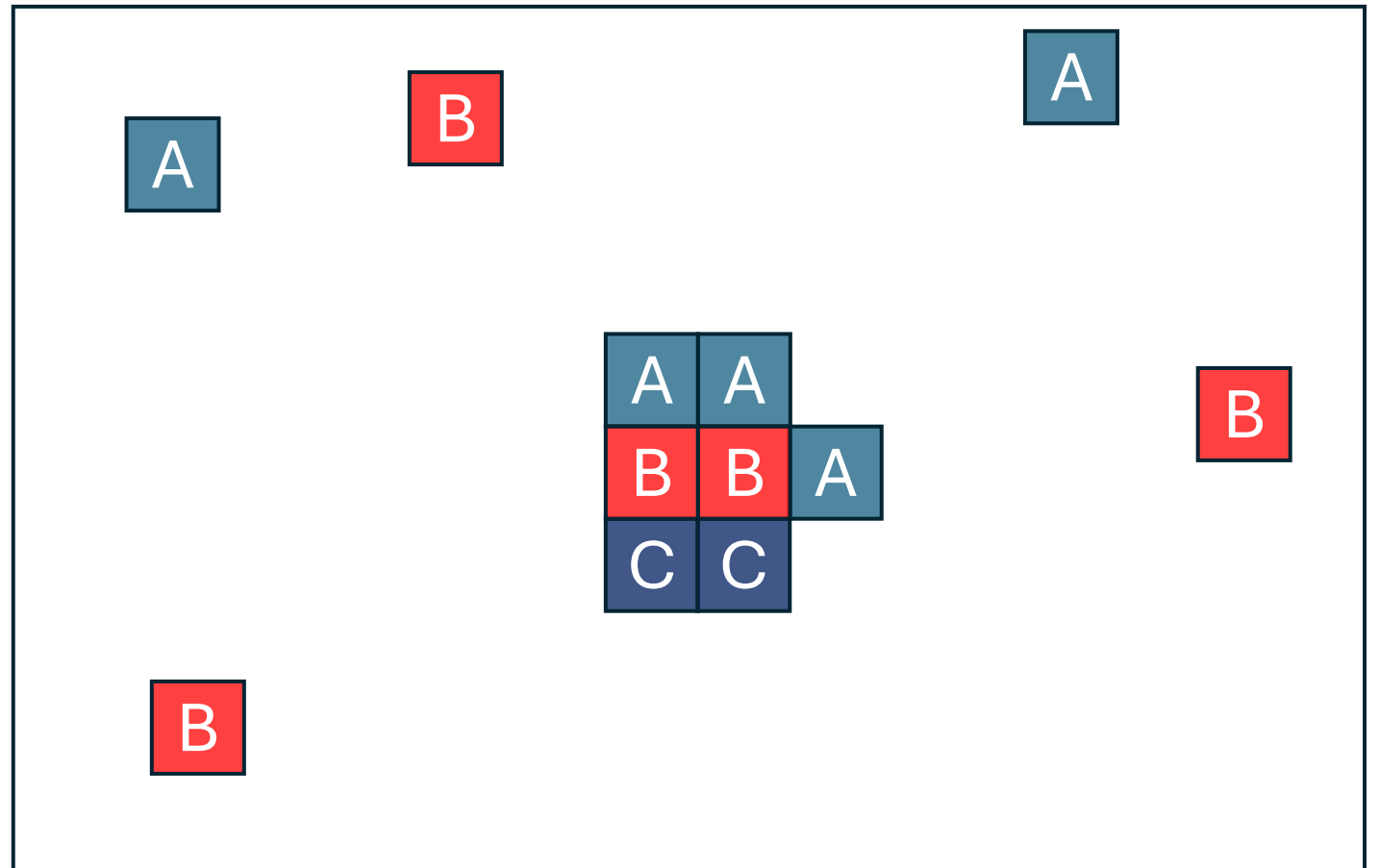


Affinities:

Temperature: τ



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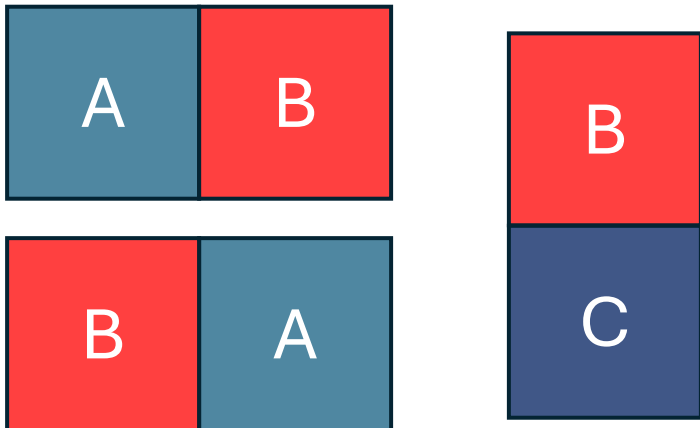
Seeded TA

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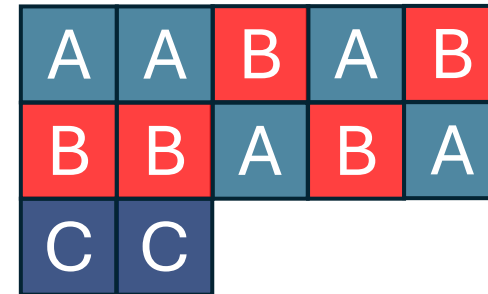


Affinities:

Temperature: τ

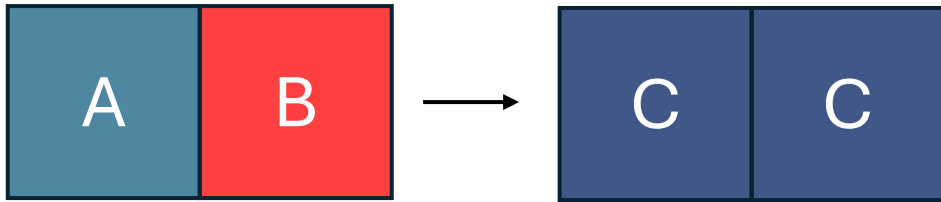


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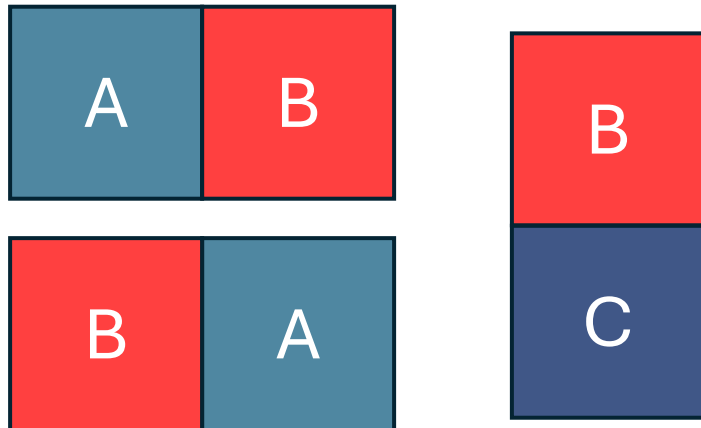
Seeded TA

Transitions:

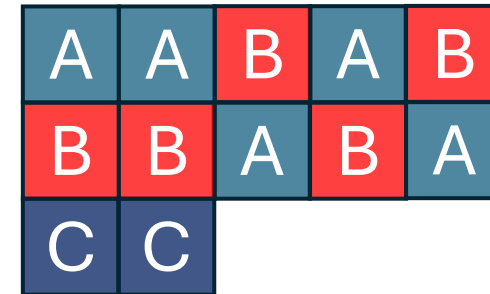


Affinities:

Temperature: τ

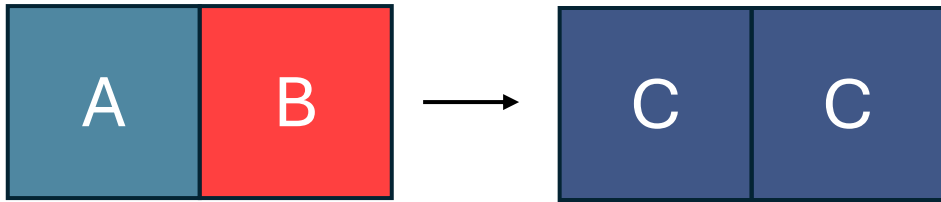


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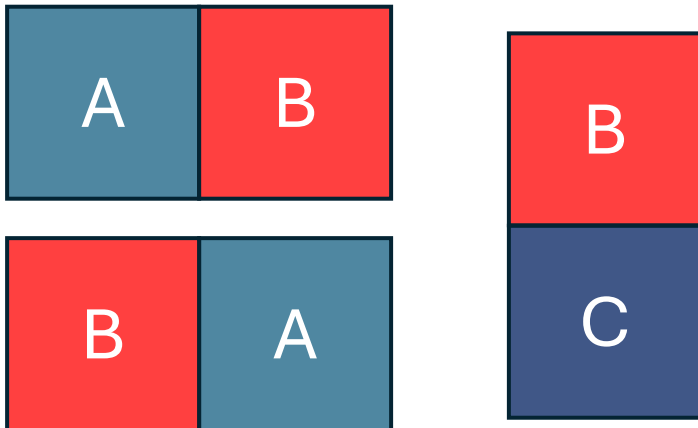
Seeded TA

Transitions:

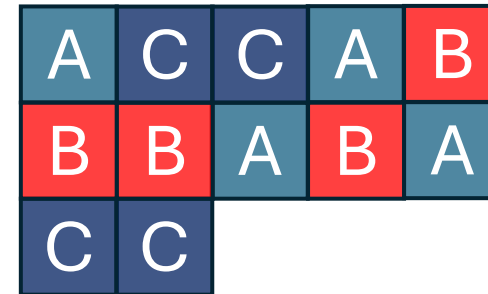


Affinities:

Temperature: τ

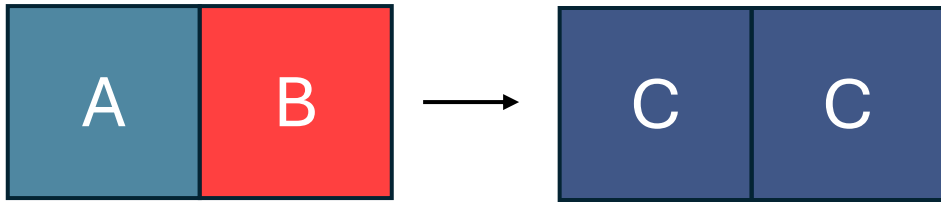


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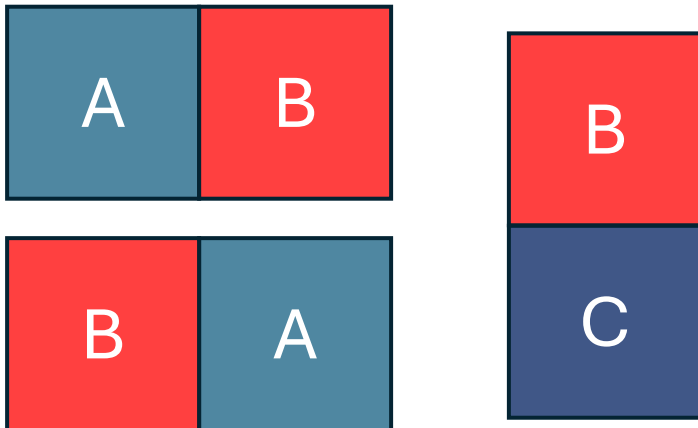
Seeded TA

Transitions:

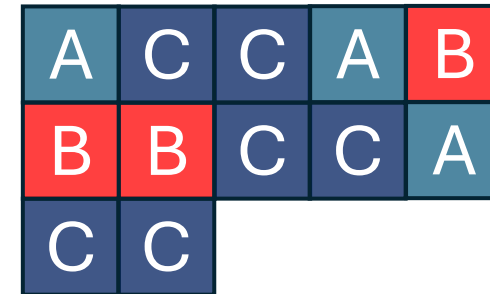


Affinities:

Temperature: τ

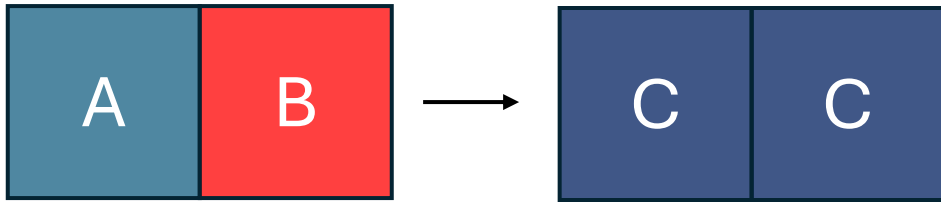


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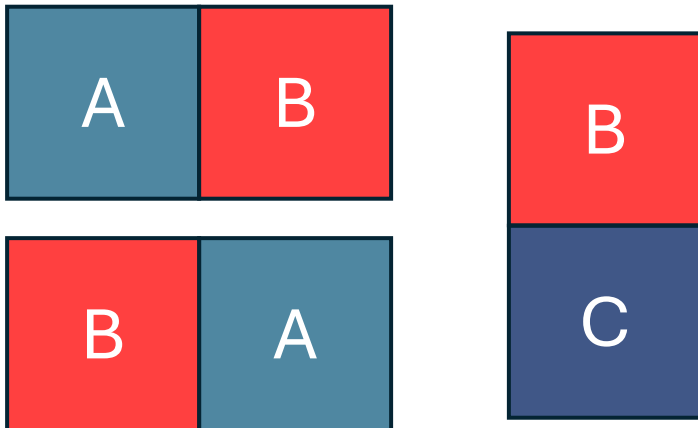
Seeded TA

Transitions:

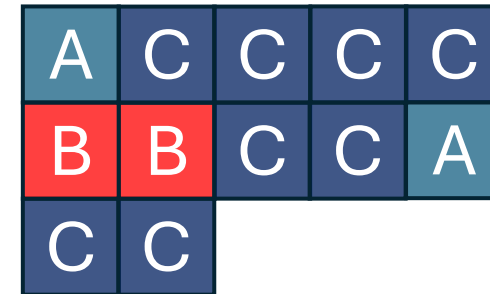


Affinities:

Temperature: τ



System:



What is known

	aTAM	2HAM	STAM	Seeded TA
Description	<ul style="list-style-type: none"> - Single tile attachments - No detachments - Fixed glues 	<ul style="list-style-type: none"> - Up to 2 assembly attachments - No detachments - Fixed glues 	<ul style="list-style-type: none"> - Up to 2 assembly attachments - Detachments - Glues turn on/off 	<ul style="list-style-type: none"> - Single tile attachments - No detachments - Transitions
Results	<ul style="list-style-type: none"> - No fractals weakly buildable (temp 1) - Some fractals not strictly buildable (temp > 1) 	<ul style="list-style-type: none"> - Finitely assemble larger class of shapes - Still some non-buildable fractals 	<ul style="list-style-type: none"> - All fractals strictly buildable - Without detachment: some impossible fractals 	<ul style="list-style-type: none"> - Fractals with generators that have a Ham-path are strictly buildable (temp 1)

Our Contribution

	aTAM	2HAM	STAM	Seeded TA
Description	<ul style="list-style-type: none">- Single tile attachments- No detachments- Fixed glues	<ul style="list-style-type: none">- Up to 2 assembly attachments- No detachments- Fixed glues	<ul style="list-style-type: none">- Up to 2 assembly attachments- Detachments- Glues turn on/off	<ul style="list-style-type: none">- Single tile attachments- No detachments- Transitions
Results	<ul style="list-style-type: none">- No fractals weakly buildable (temp 1)- Some fractals not strictly buildable (temp > 1)	<ul style="list-style-type: none">- Finitely assemble larger class of shapes- Still some non-buildable fractals	<ul style="list-style-type: none">- All fractals strictly buildable- Without detachment: some impossible fractals	<ul style="list-style-type: none">- Fractals with generators that have a Ham-path are strictly buildable (temp 1)

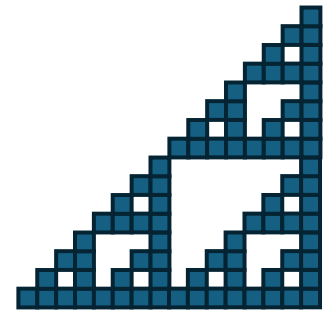
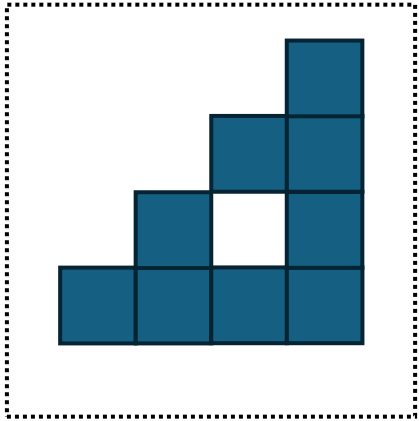
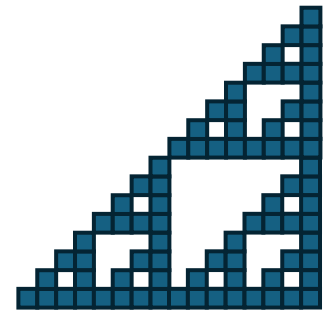
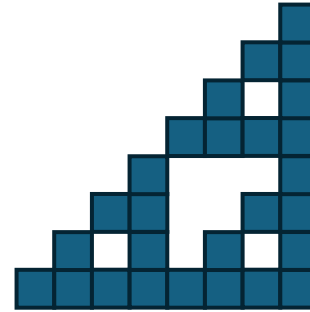
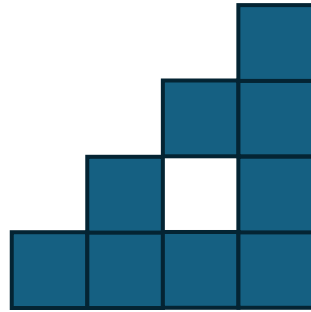
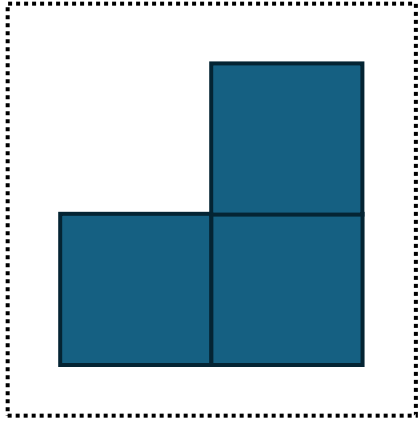
Our Contribution

	aTAM	2HAM	STAM	Seeded TA
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Results	<ul style="list-style-type: none"> - No fractals weakly buildable (temp 1) - Some fractals not strictly buildable (temp > 1) 	<ul style="list-style-type: none"> - Finitely assemble larger class of shapes - Still some non-buildable fractals 	<ul style="list-style-type: none"> - All fractals strictly buildable - Without detachment: some impossible fractals 	<ul style="list-style-type: none"> - ALL fractals are strictly buildable (temp 1) - There exists a single system that does so

The Construction

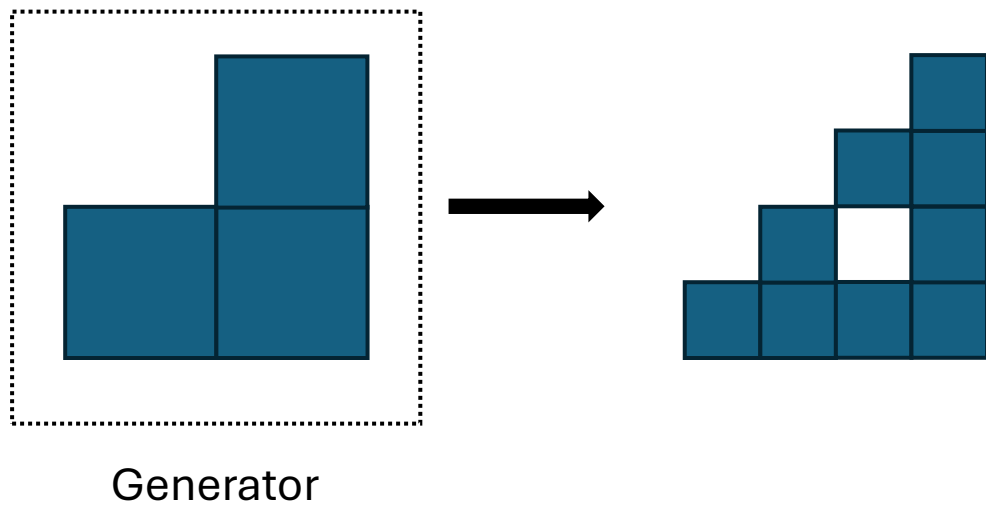
How did we do it?

Important Observation

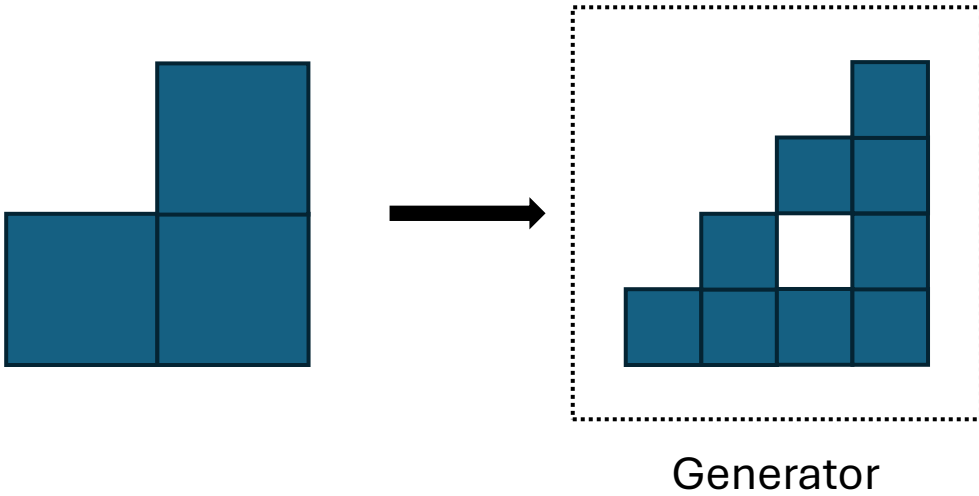


Generators

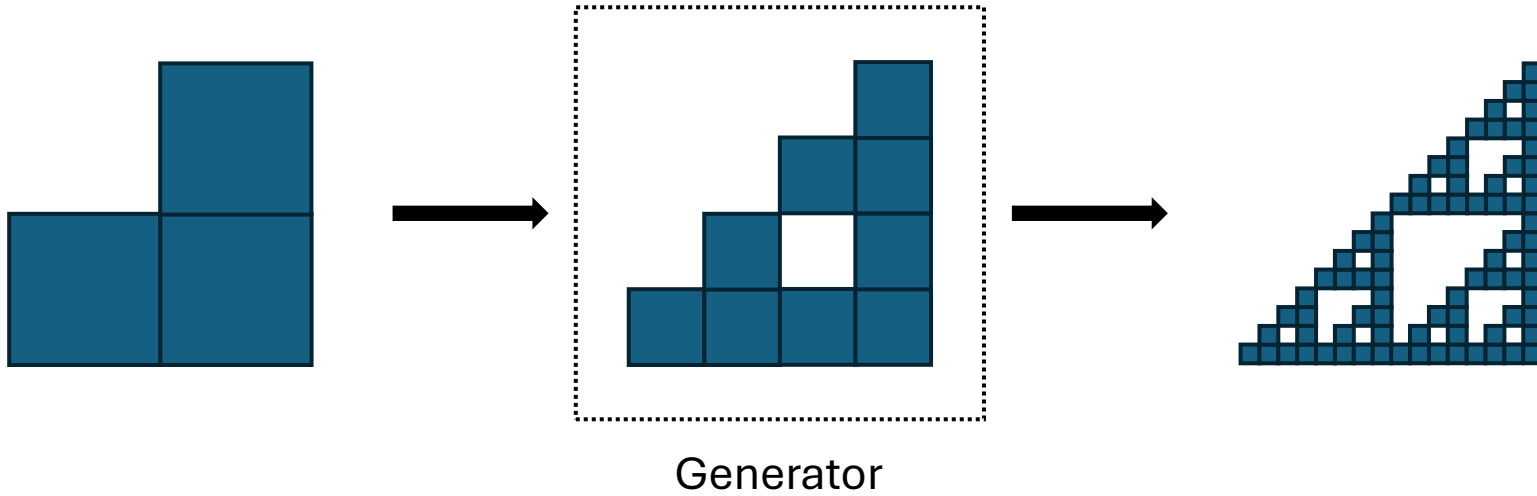
Important Observation



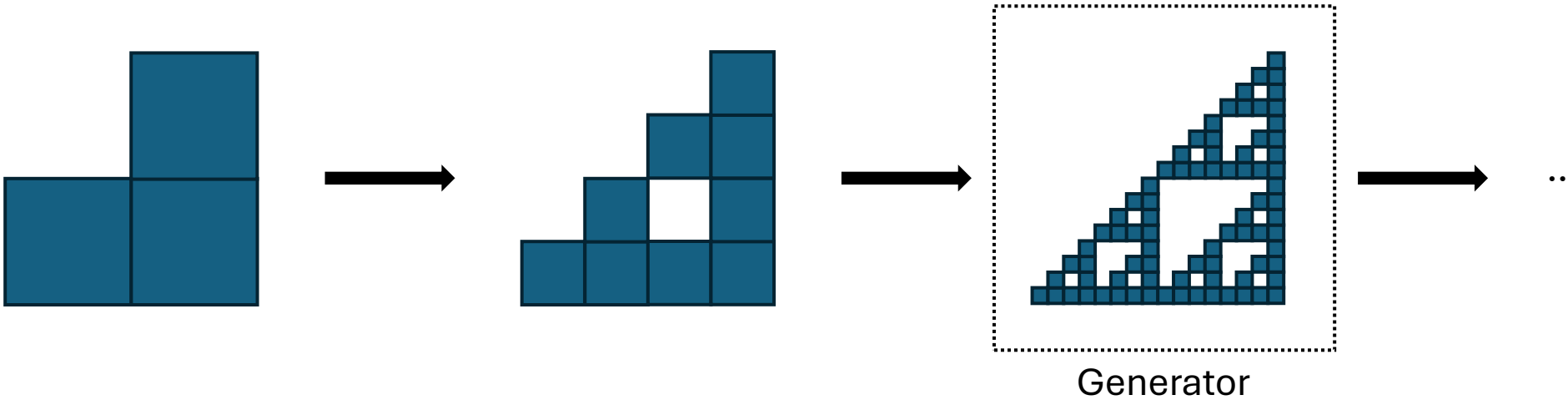
Important Observation



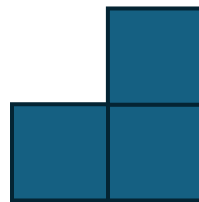
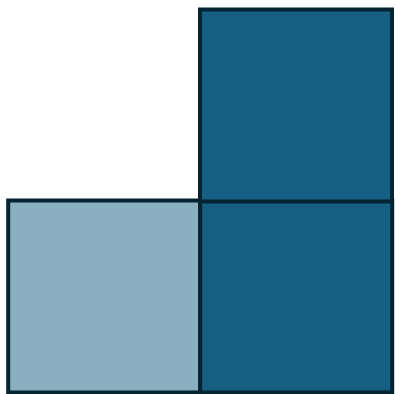
Important Observation



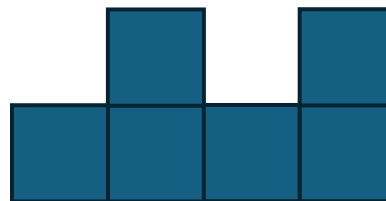
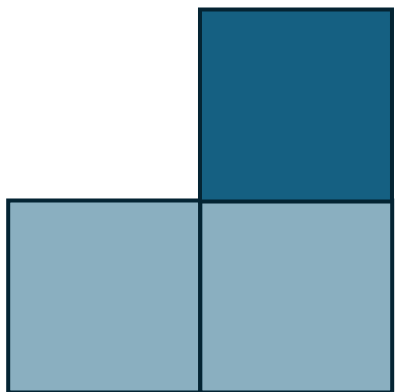
Important Observation



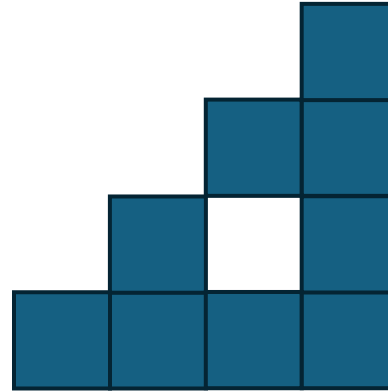
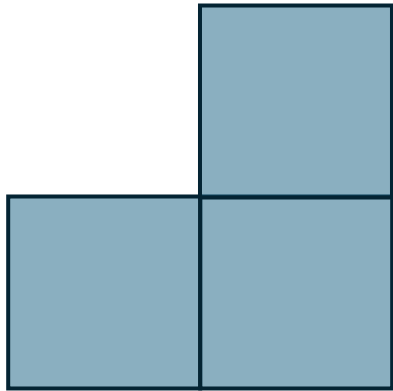
High Level Idea



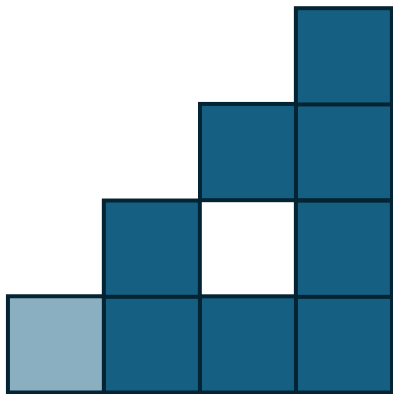
High Level Idea



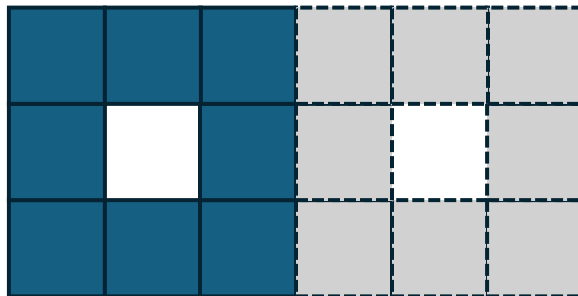
High Level Idea



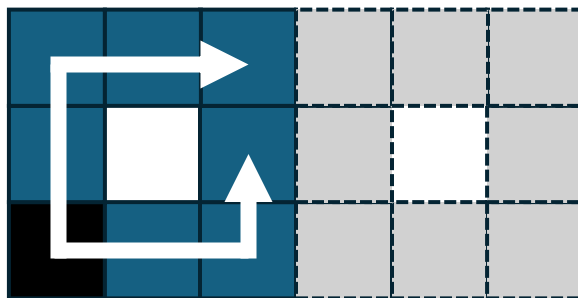
High Level Idea



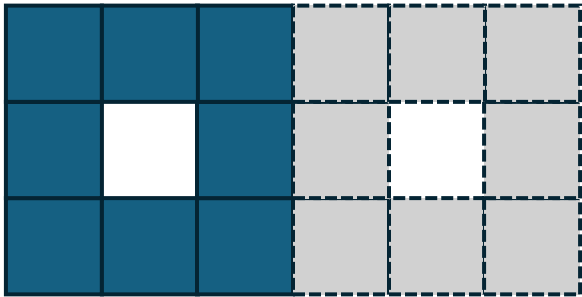
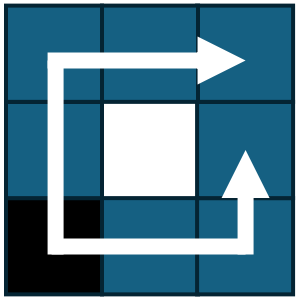
Step 1: Devising an Order



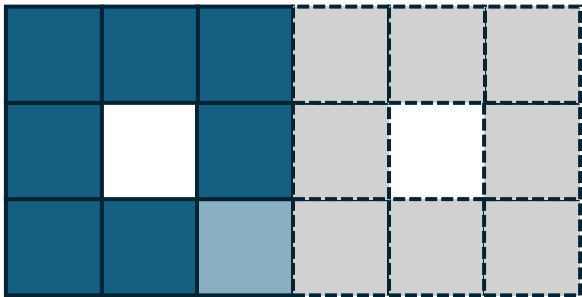
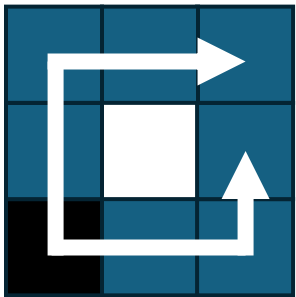
Step 1: Devising an Order



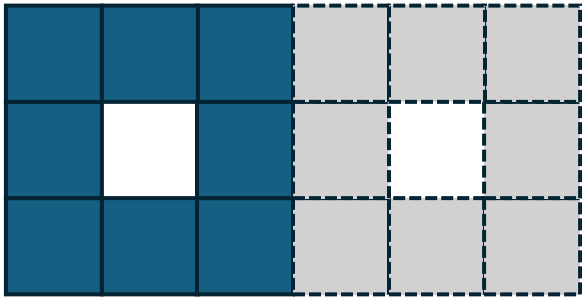
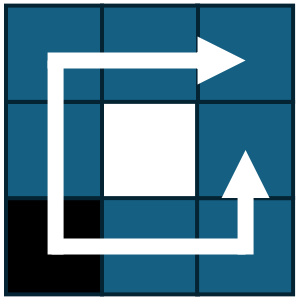
Step 2: A Copying Procedure



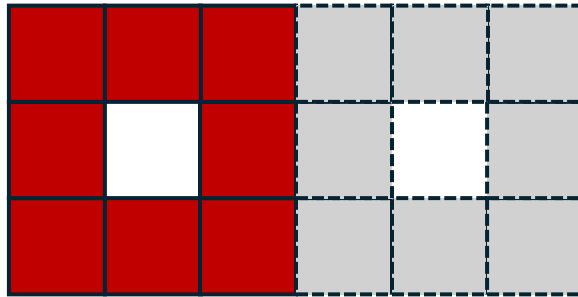
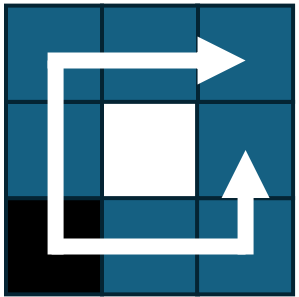
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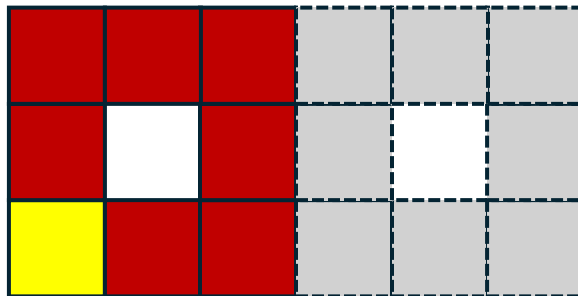
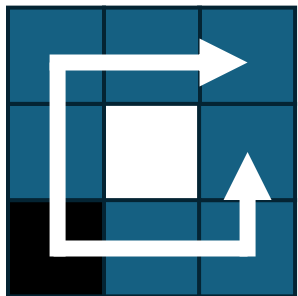
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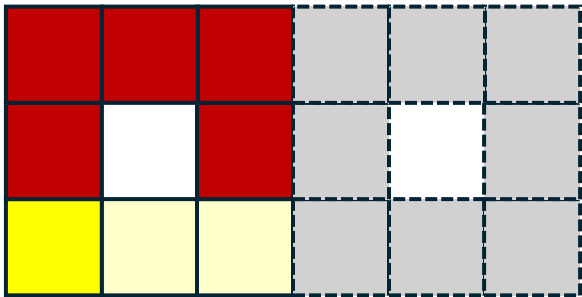
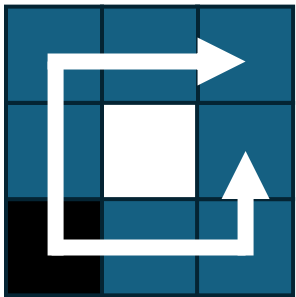
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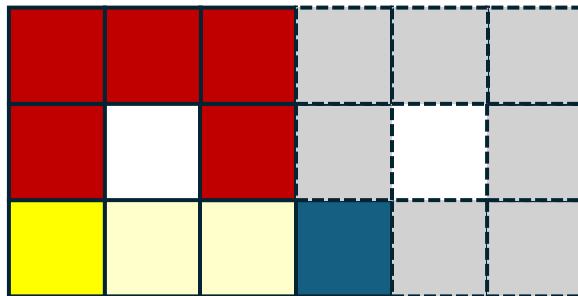
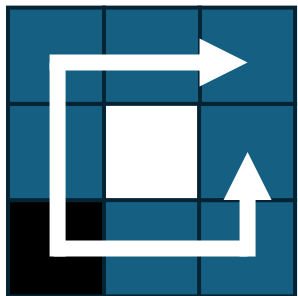
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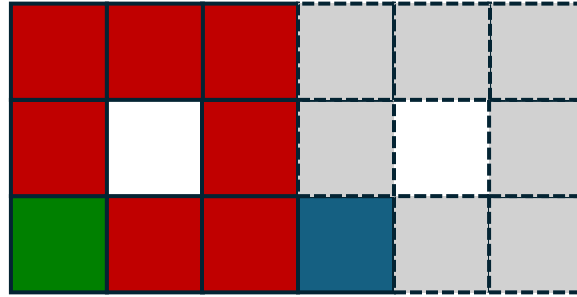
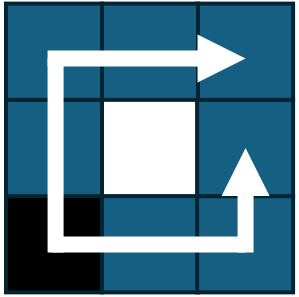
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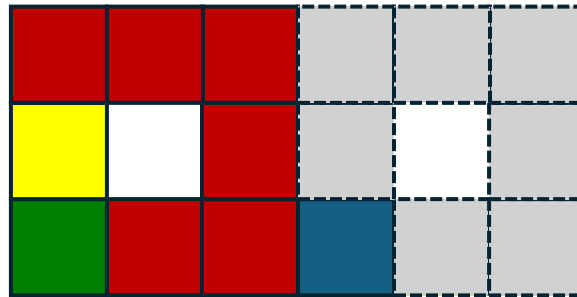
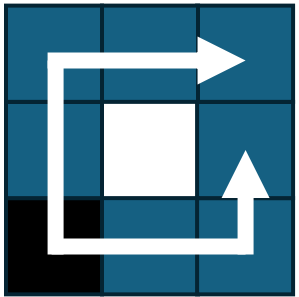
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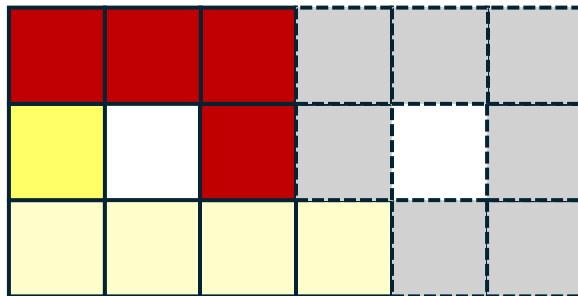
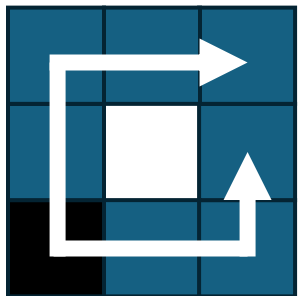
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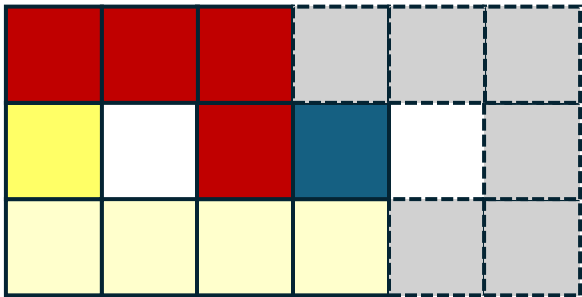
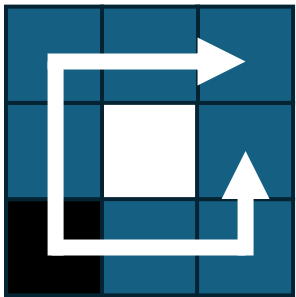
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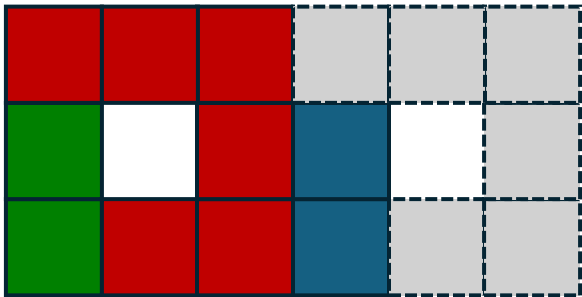
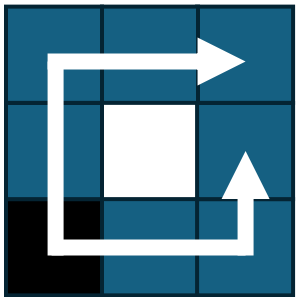
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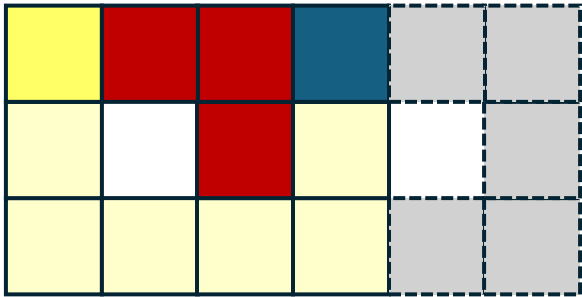
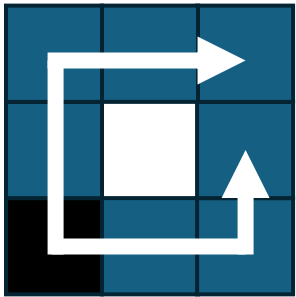
Step 2: A Copying Procedure



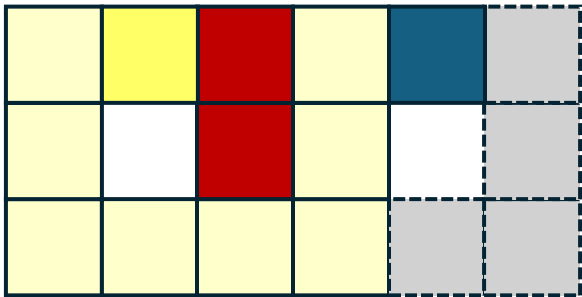
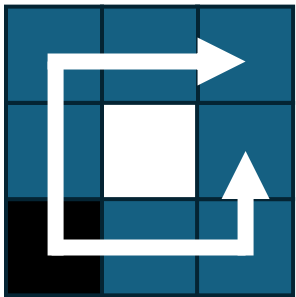
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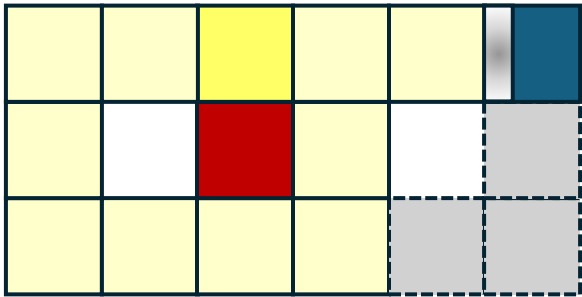
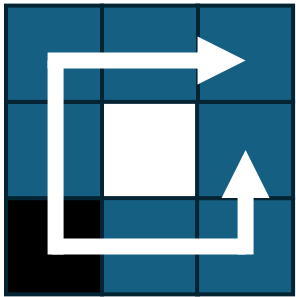
Step 2: A Copying Procedure



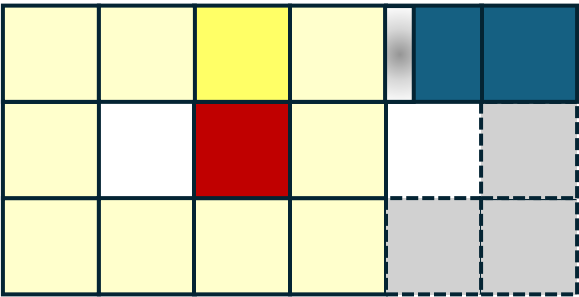
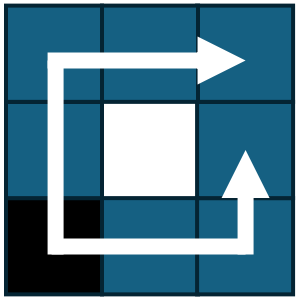
Step 2: A Copying Procedure



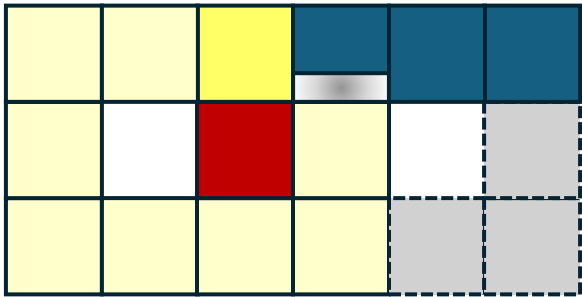
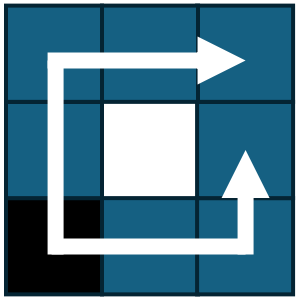
Step 2: A Copying Procedure



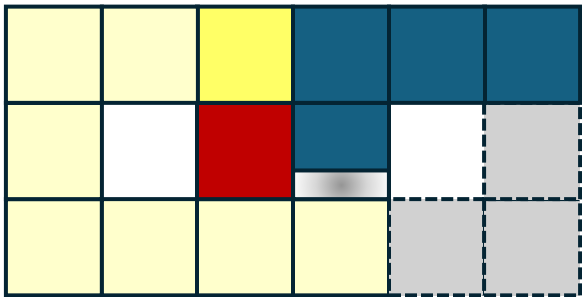
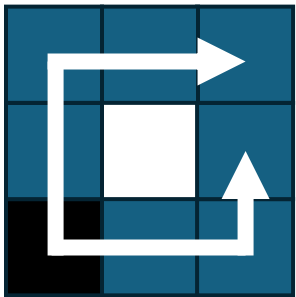
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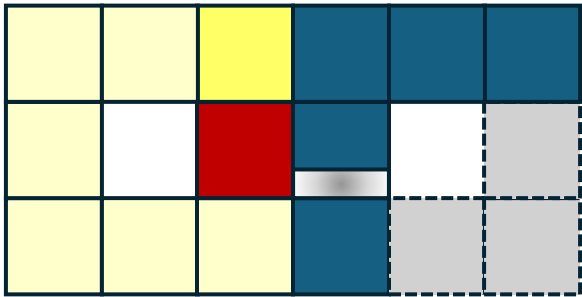
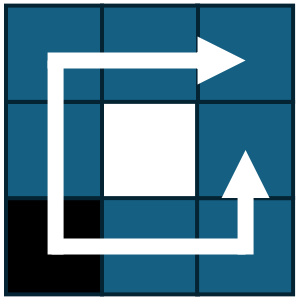
Step 2: A Copying Procedure



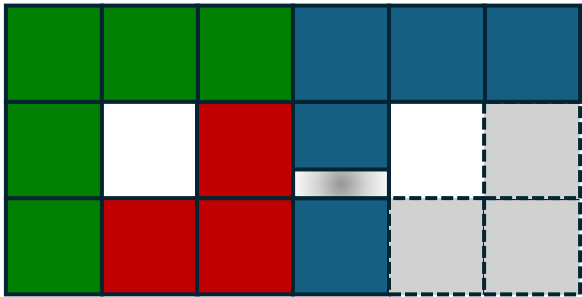
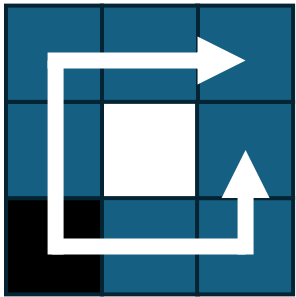
Step 2: A Copying Procedure



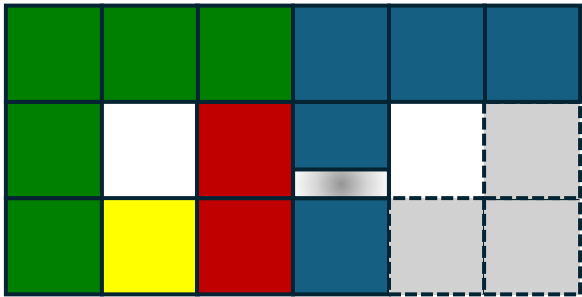
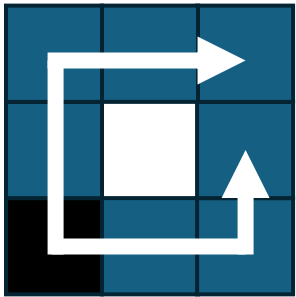
Step 2: A Copying Procedure



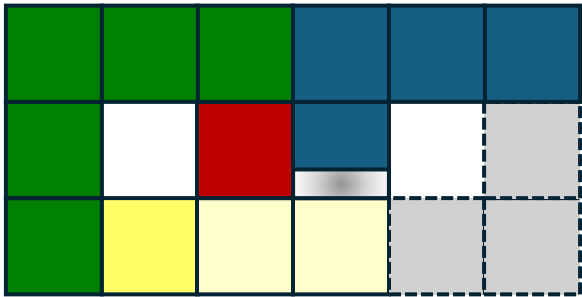
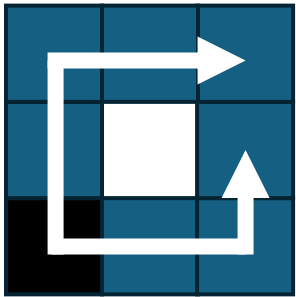
Step 2: A Copying Procedure



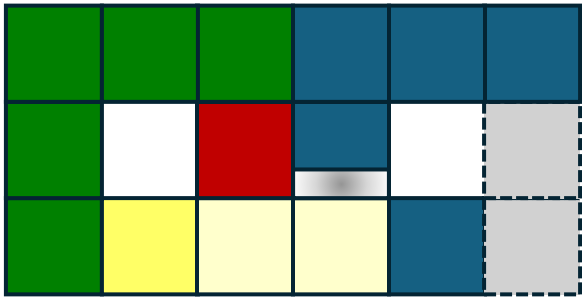
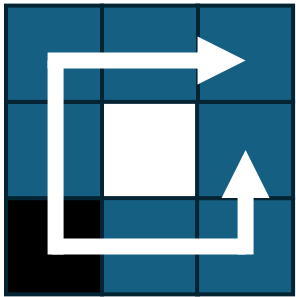
Step 2: A Copying Procedure



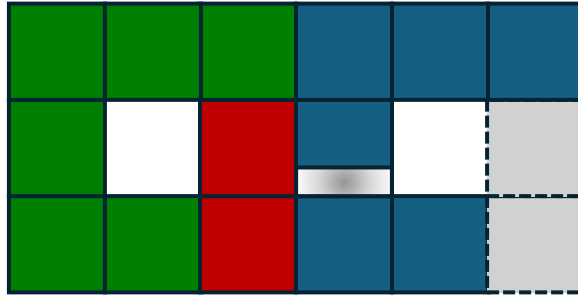
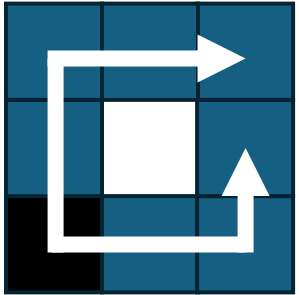
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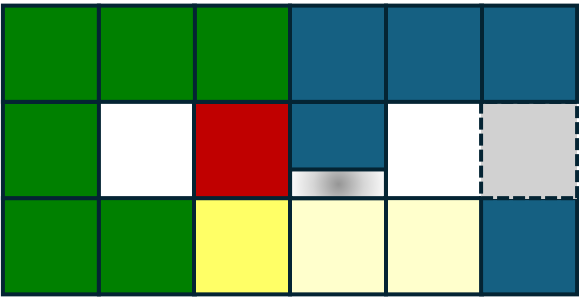
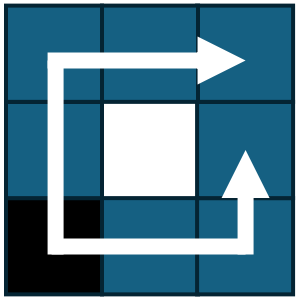
Step 2: A Copying Procedure



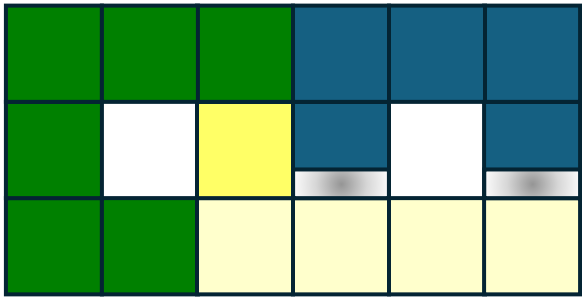
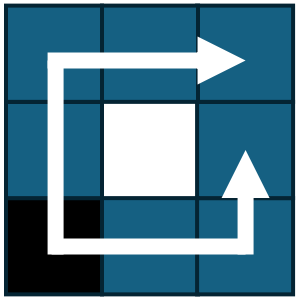
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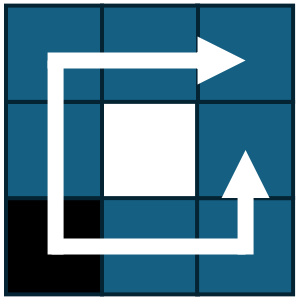
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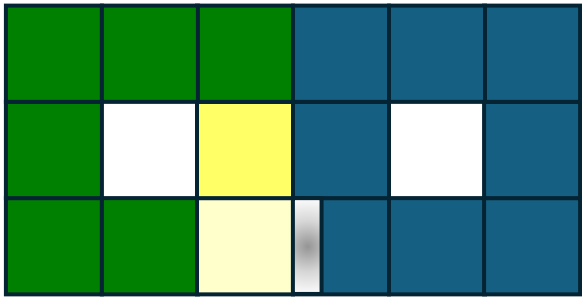
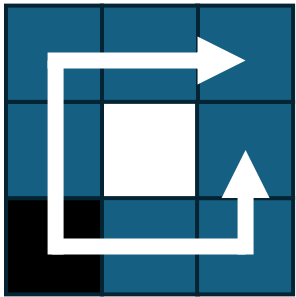
Step 2: A Copying Procedure



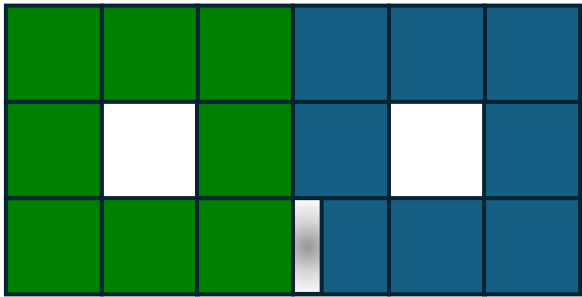
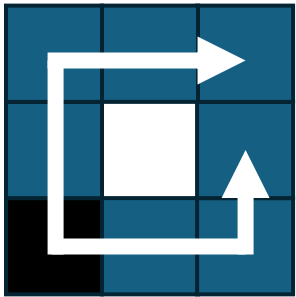
Step 2: A Copying Procedure



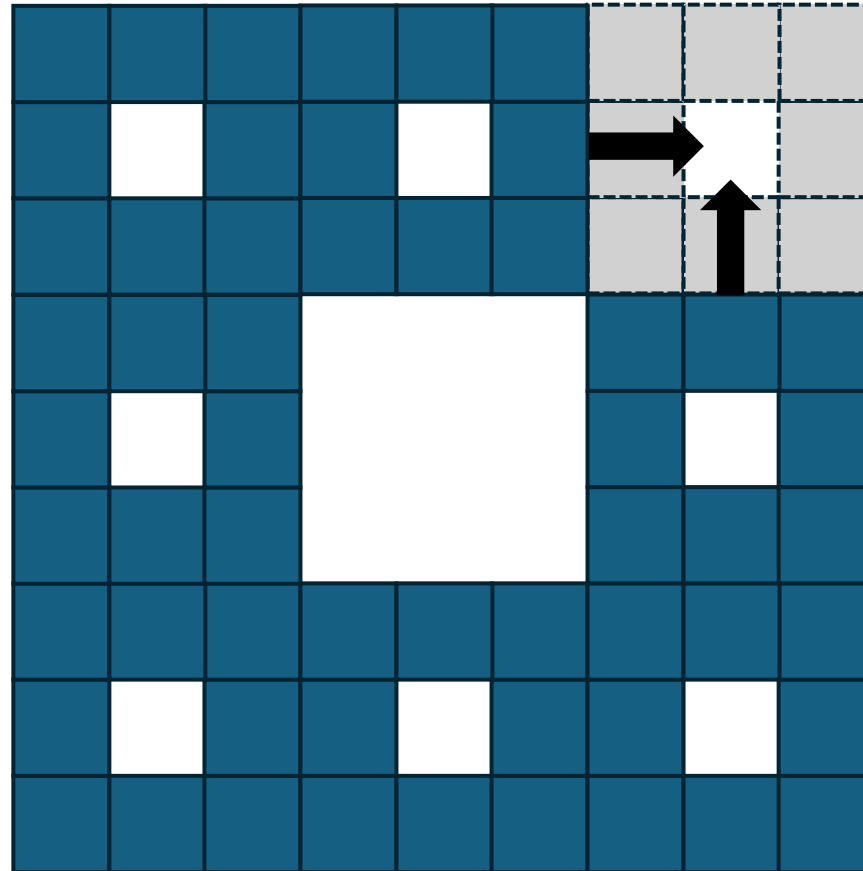
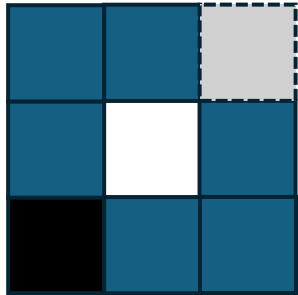
Step 2: A Copying Procedure



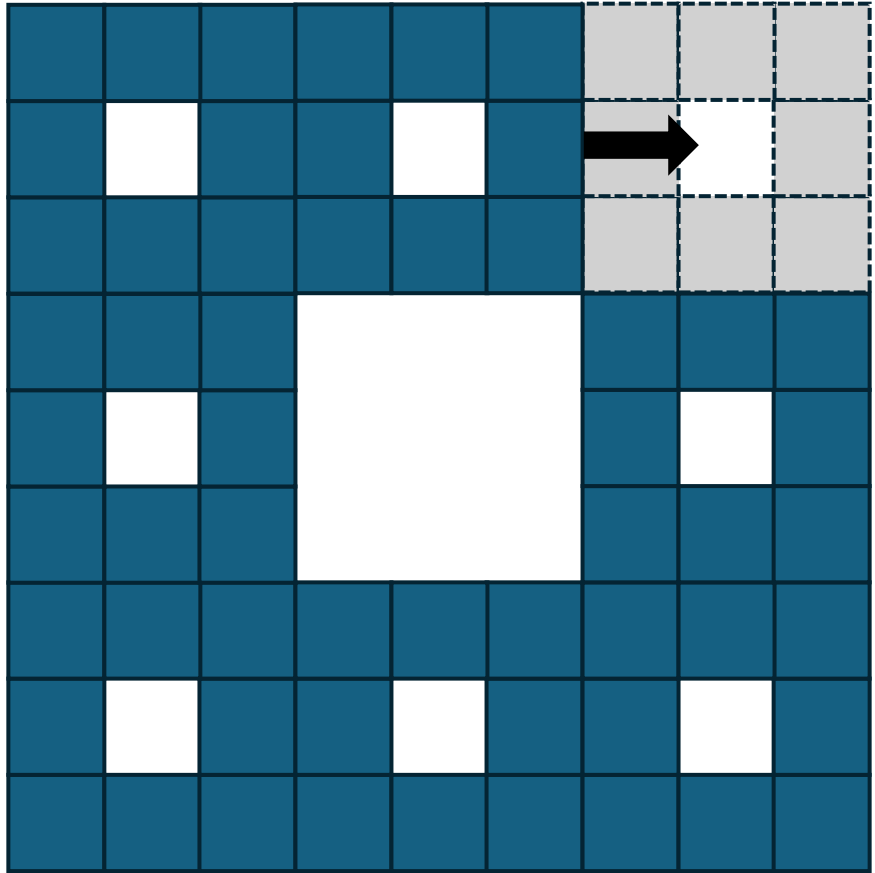
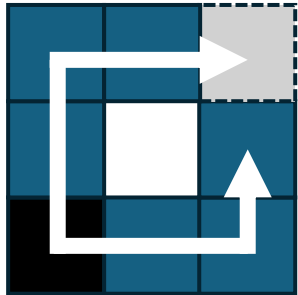
Step 2: A Copying Procedure



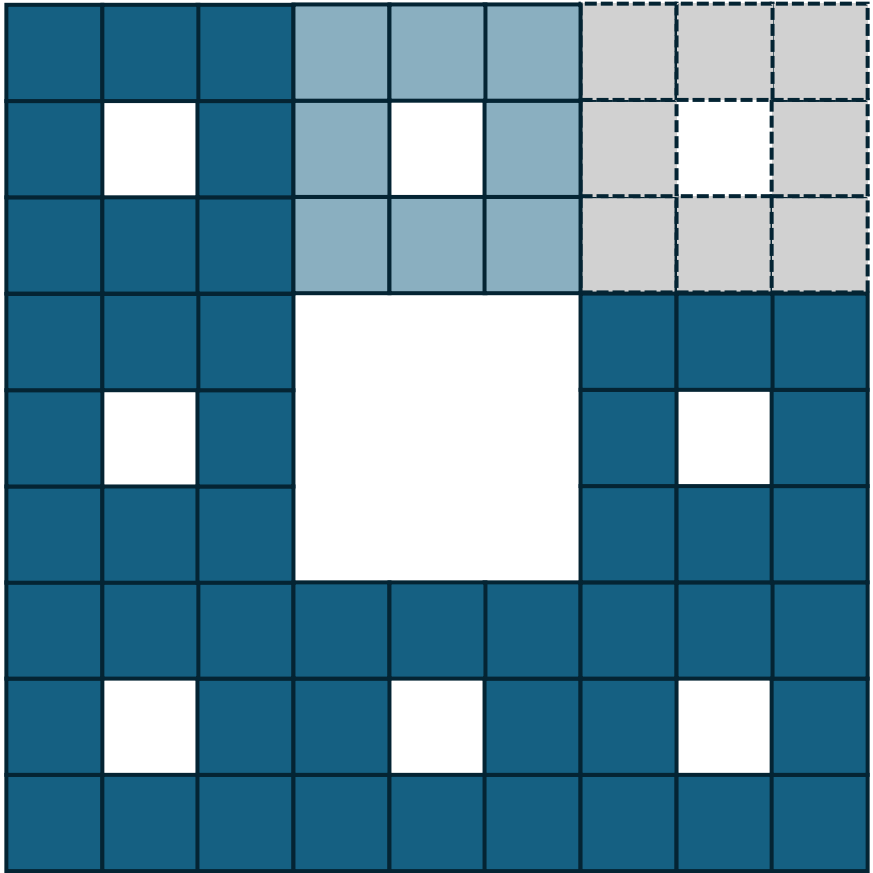
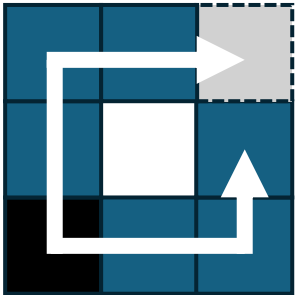
Step 3: Growing the Full Assembly



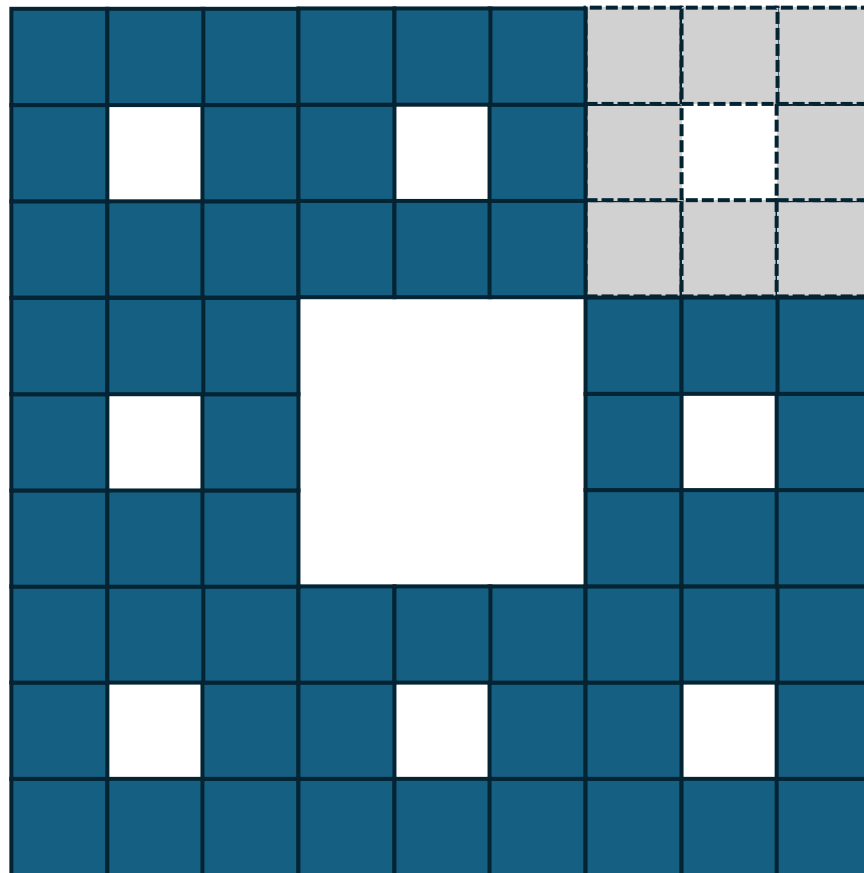
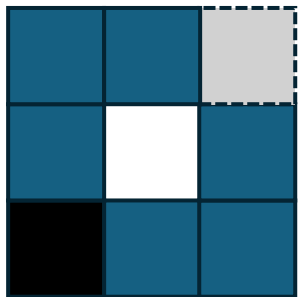
Step 3: Growing the Full Assembly



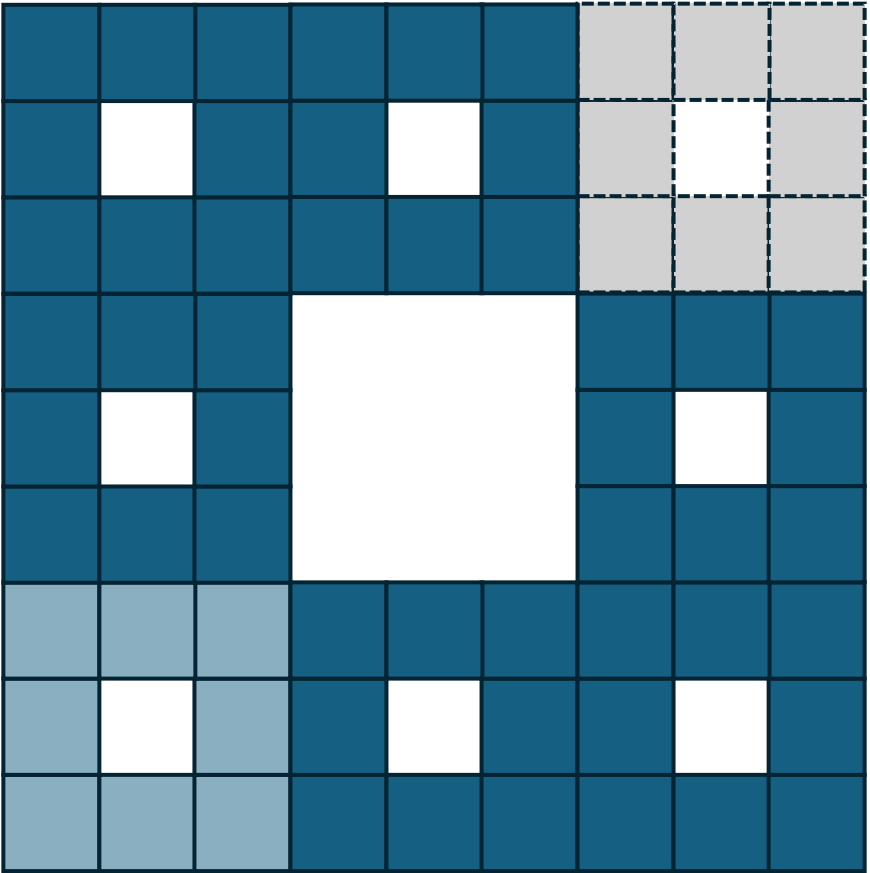
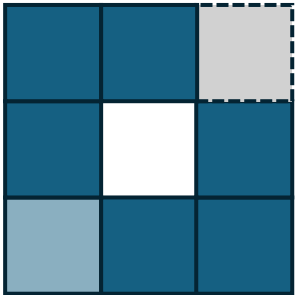
Step 3: Growing the Full Assembly



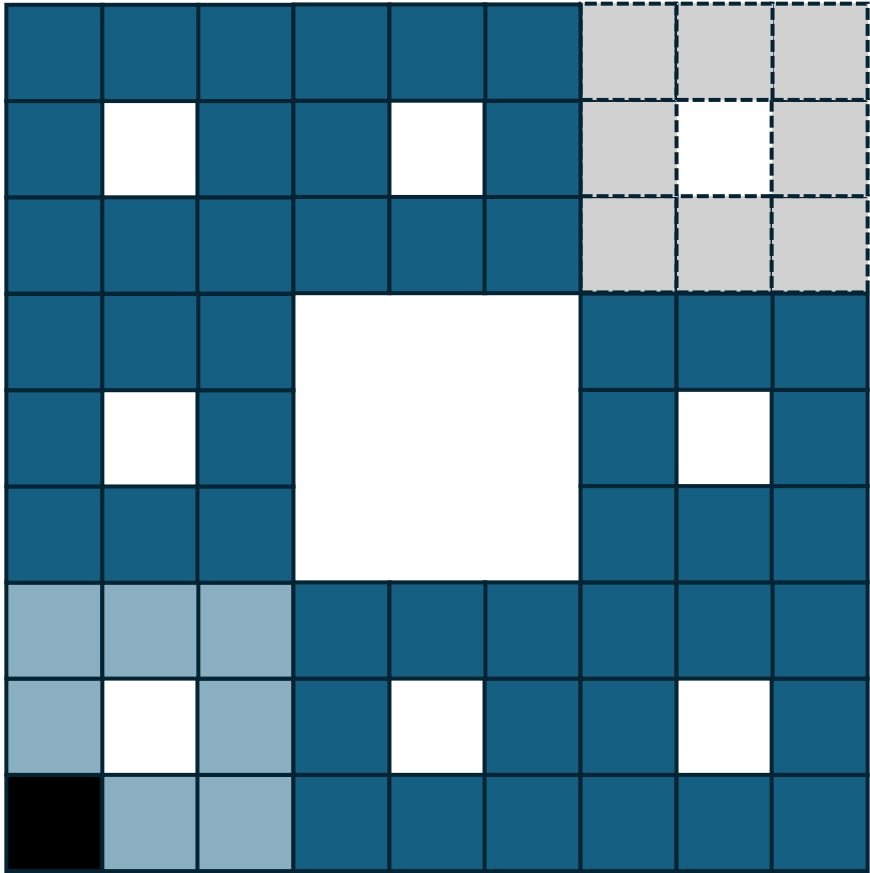
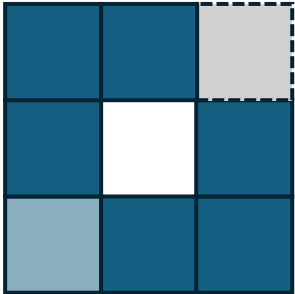
Step 3: Growing the Full Assembly



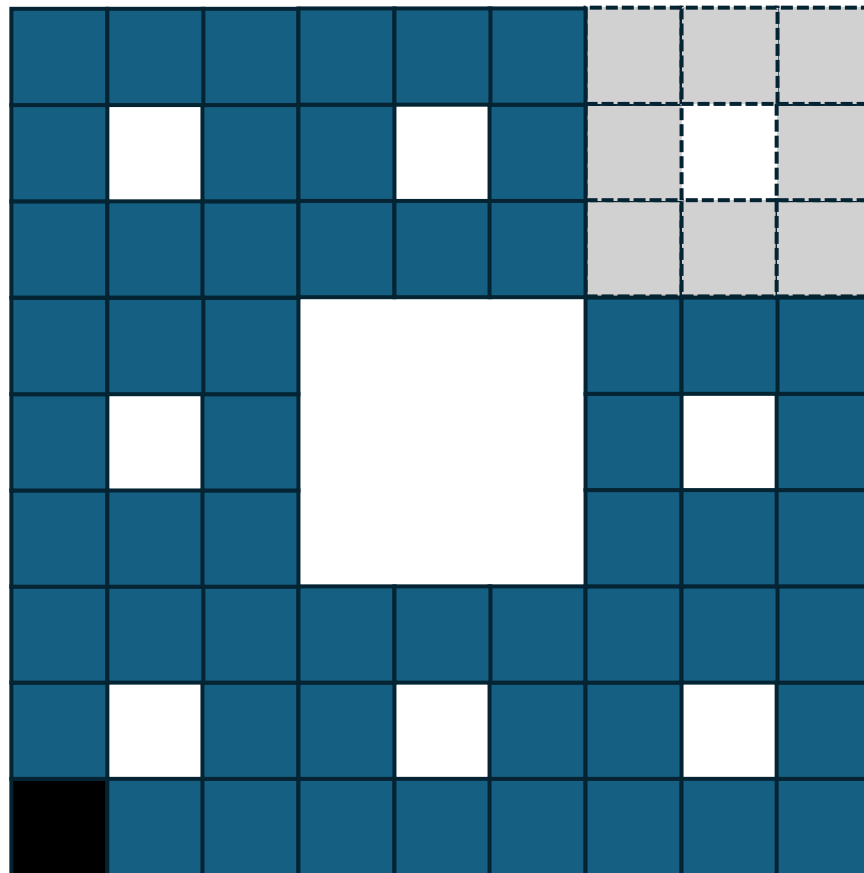
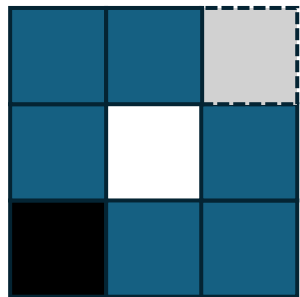
Step 3: Growing the Full Assembly



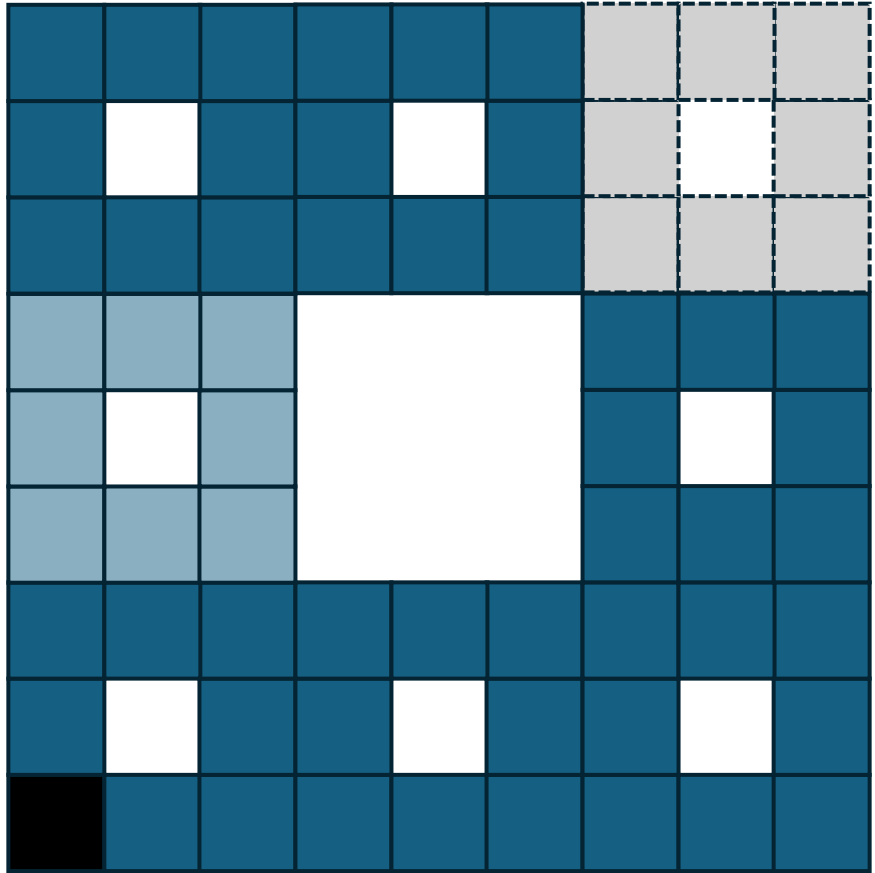
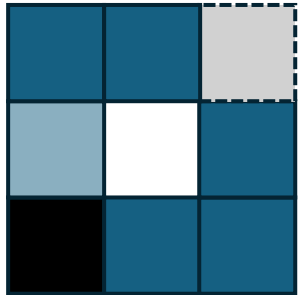
Step 3: Growing the Full Assembly



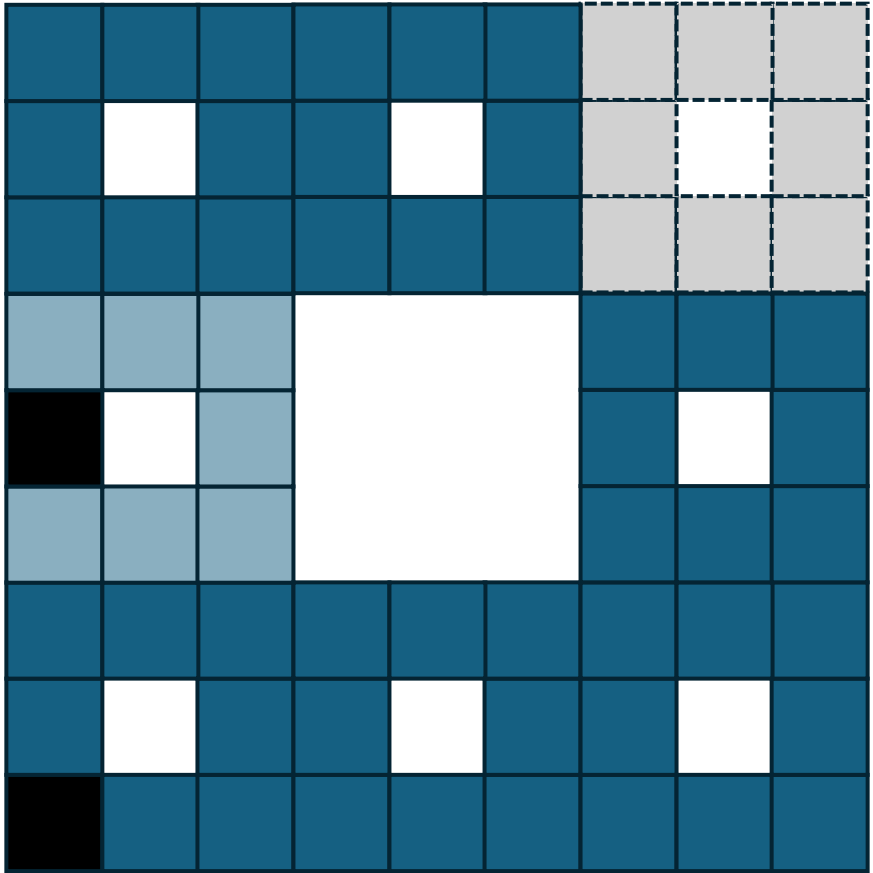
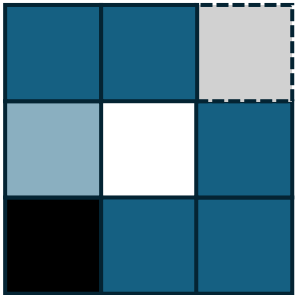
Step 3: Growing the Full Assembly



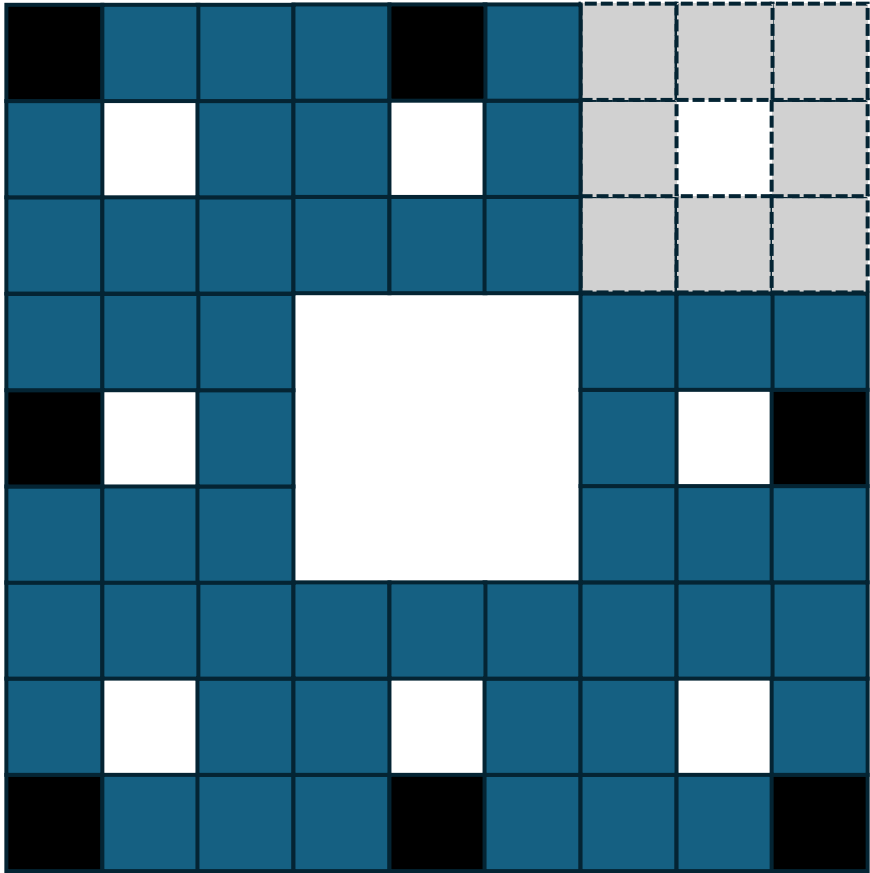
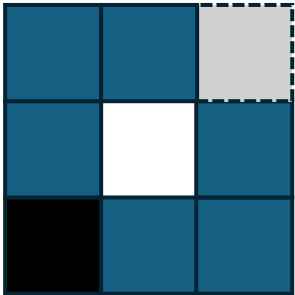
Step 3: Growing the Full Assembly



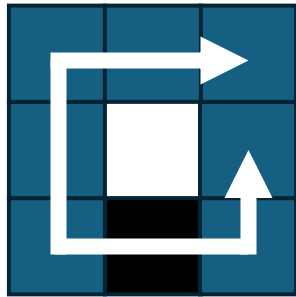
Step 3: Growing the Full Assembly



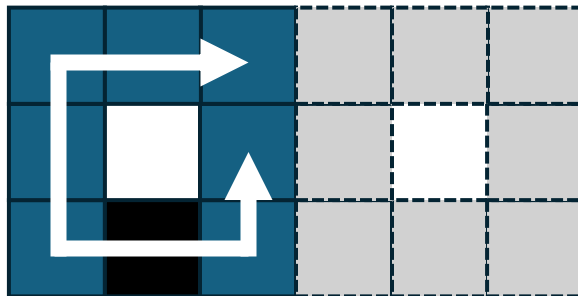
Step 3: Growing the Full Assembly



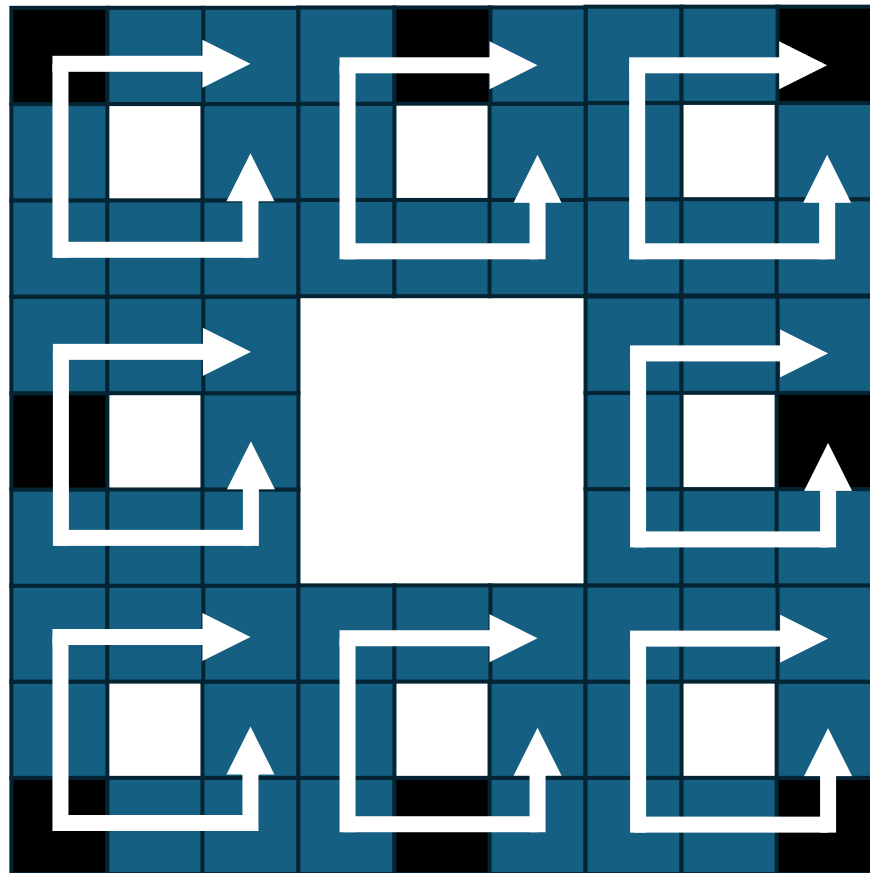
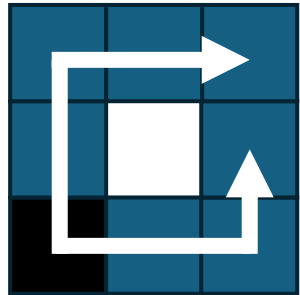
Step 3: Growing the Full Assembly



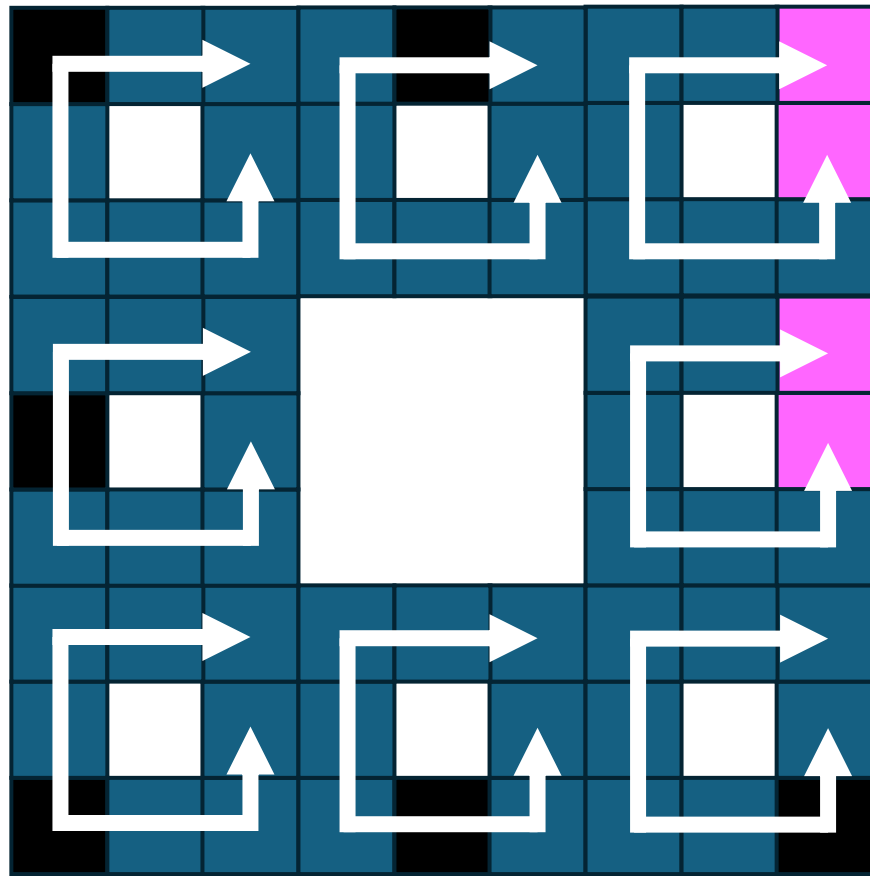
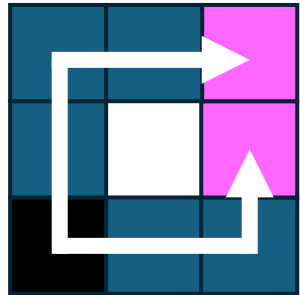
Step 3: Growing the Full Assembly



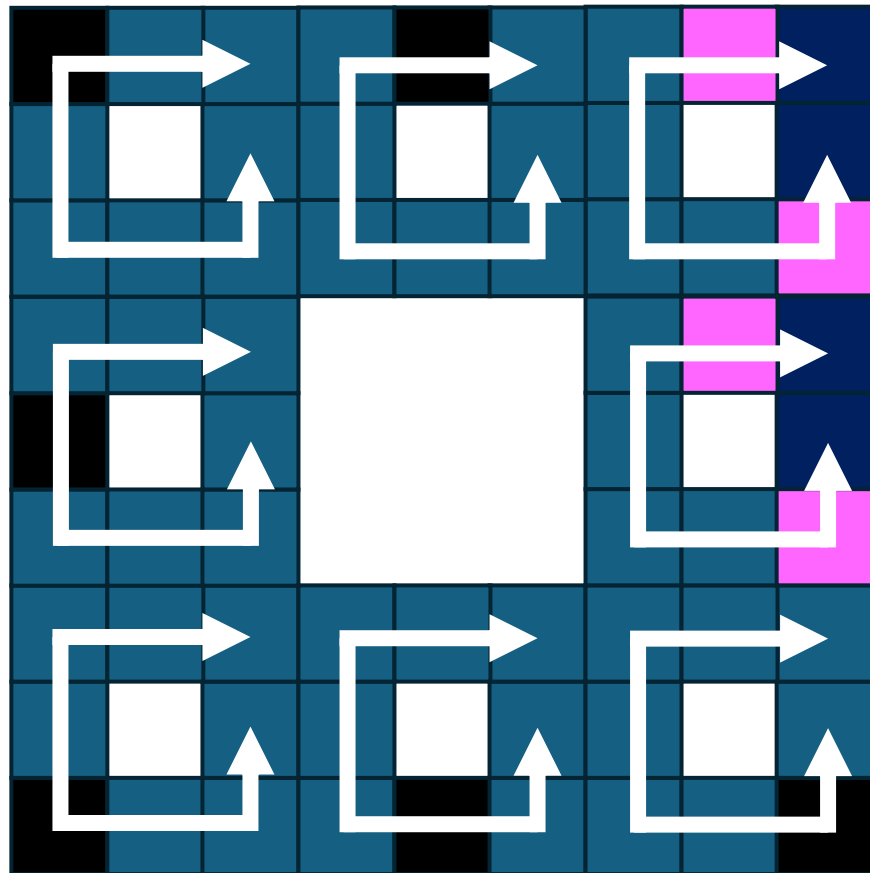
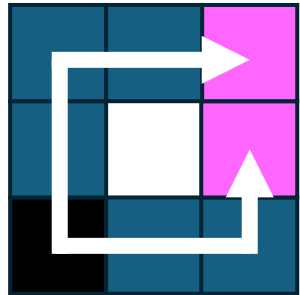
Step 4: Resetting



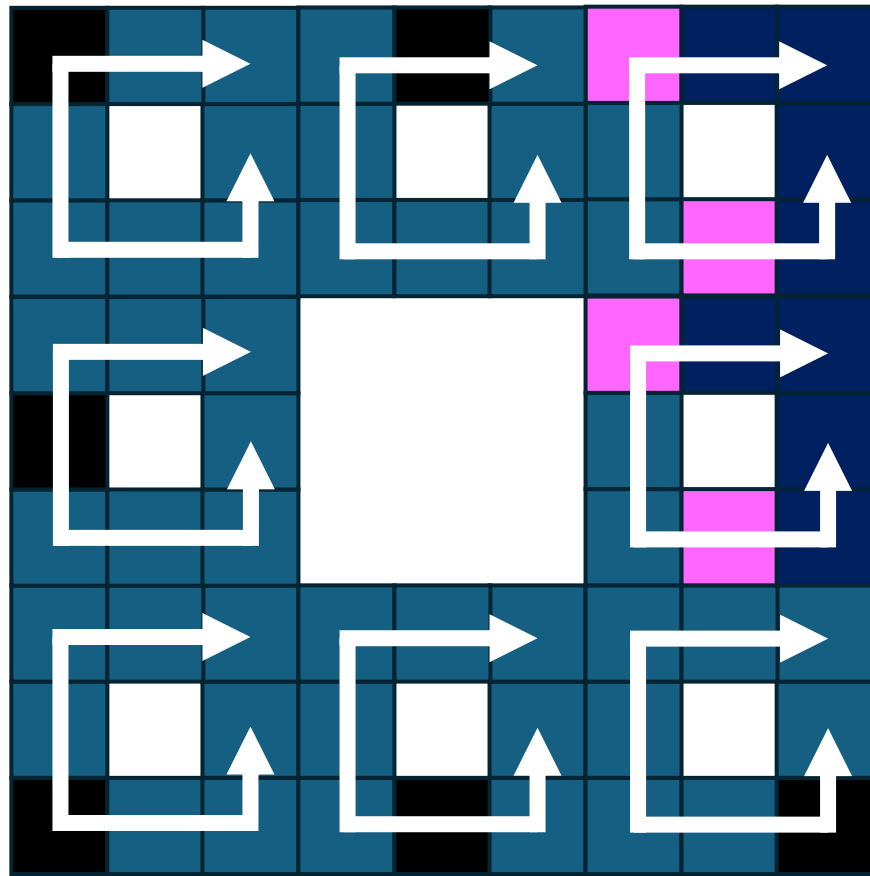
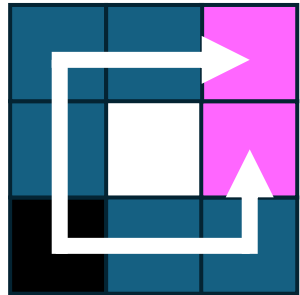
Step 4: Resetting



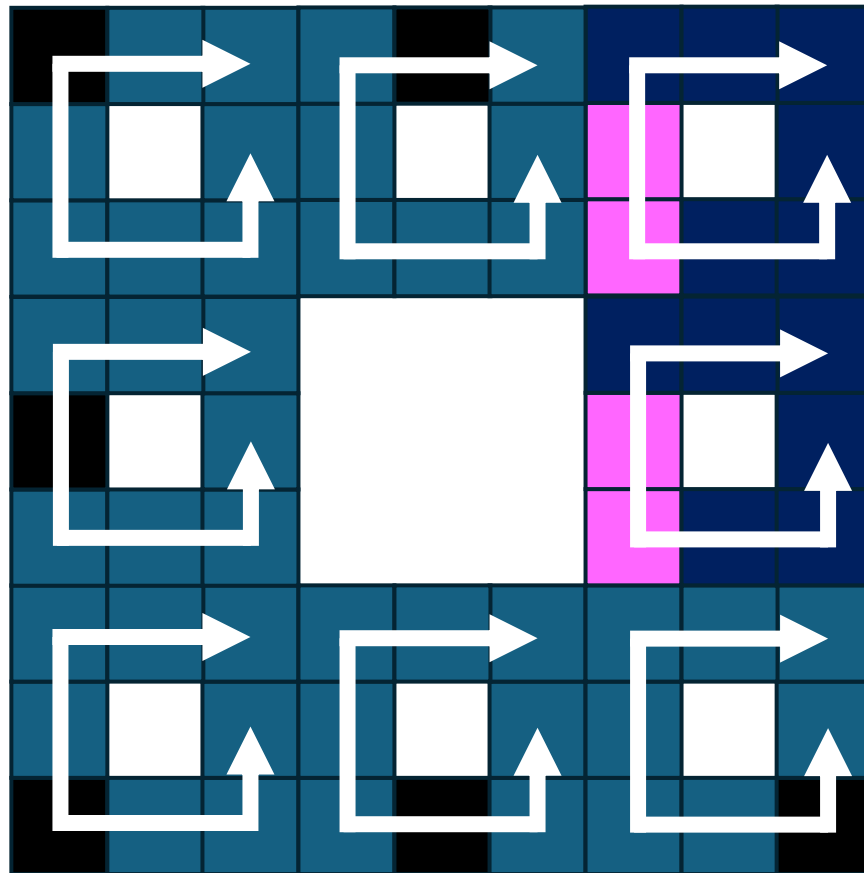
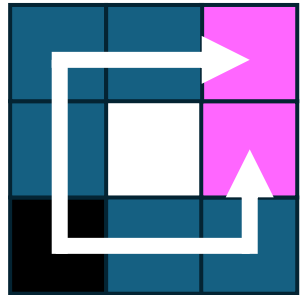
Step 4: Resetting



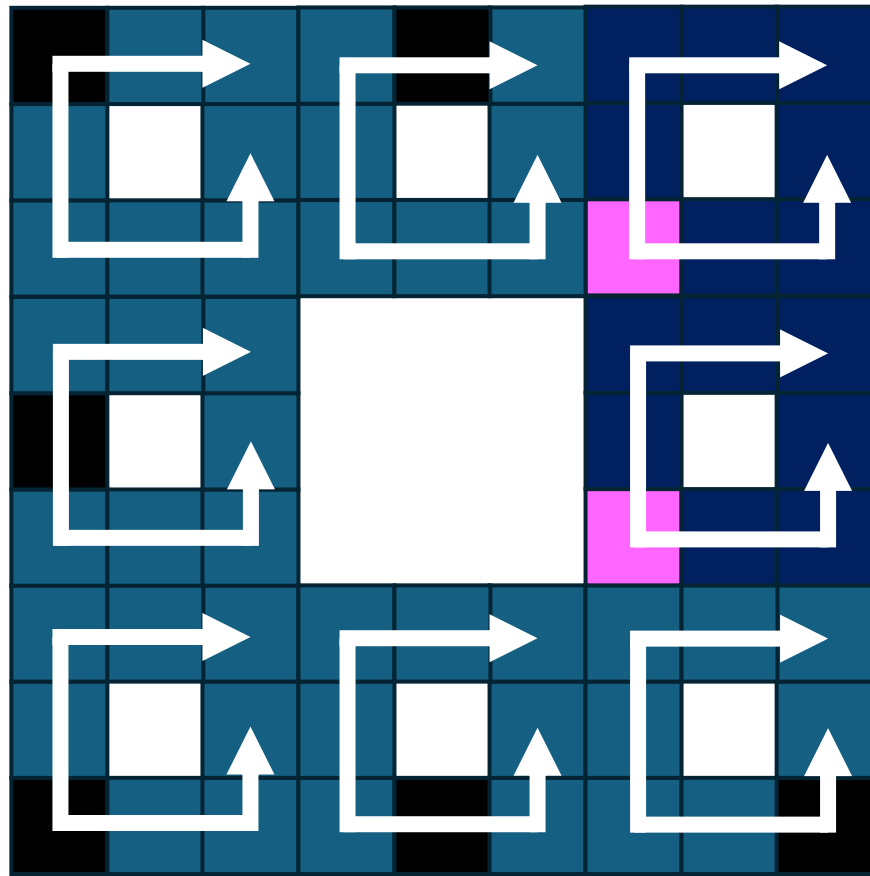
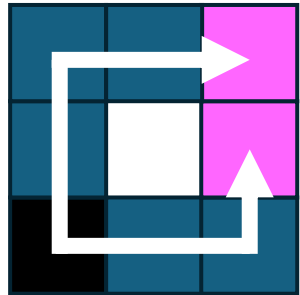
Step 4: Resetting



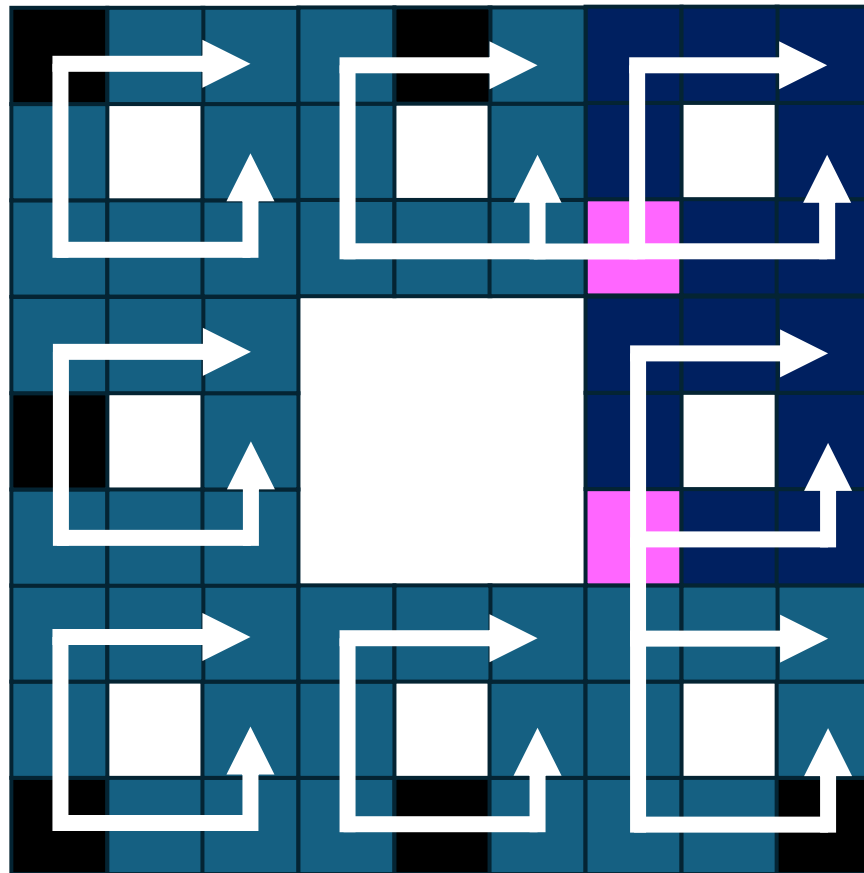
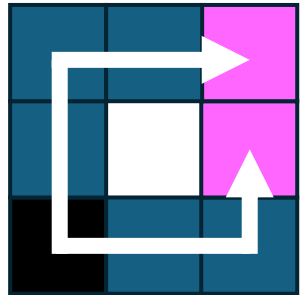
Step 4: Resetting



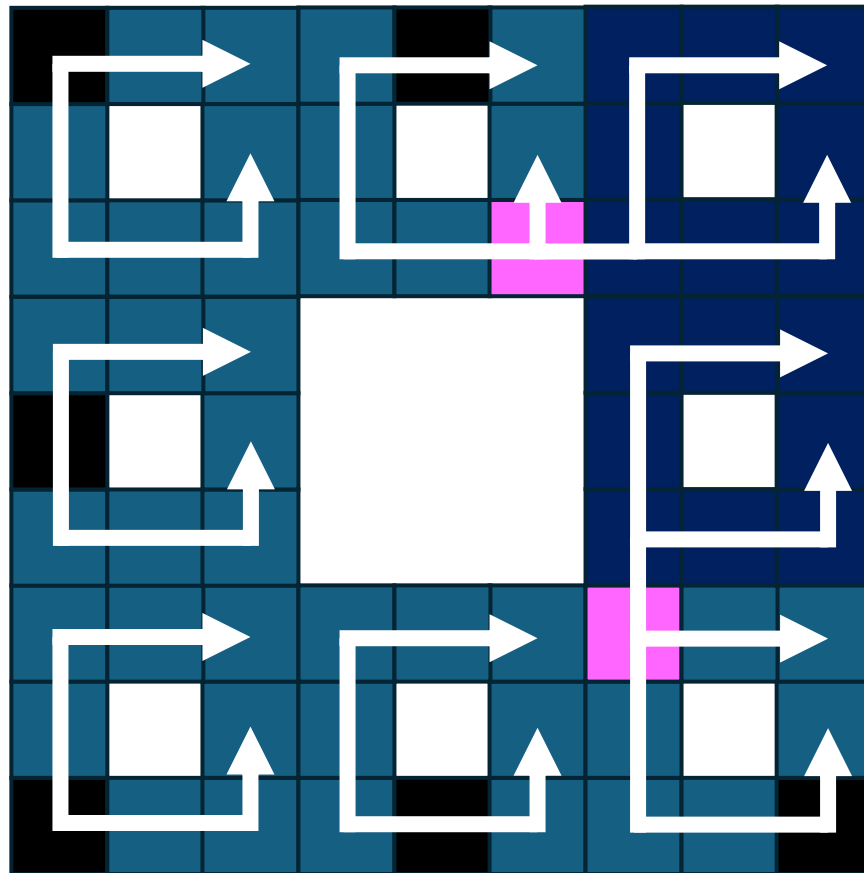
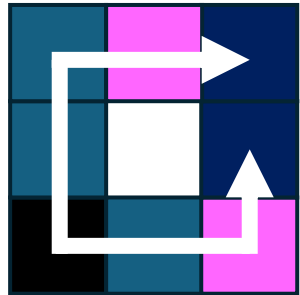
Step 4: Resetting



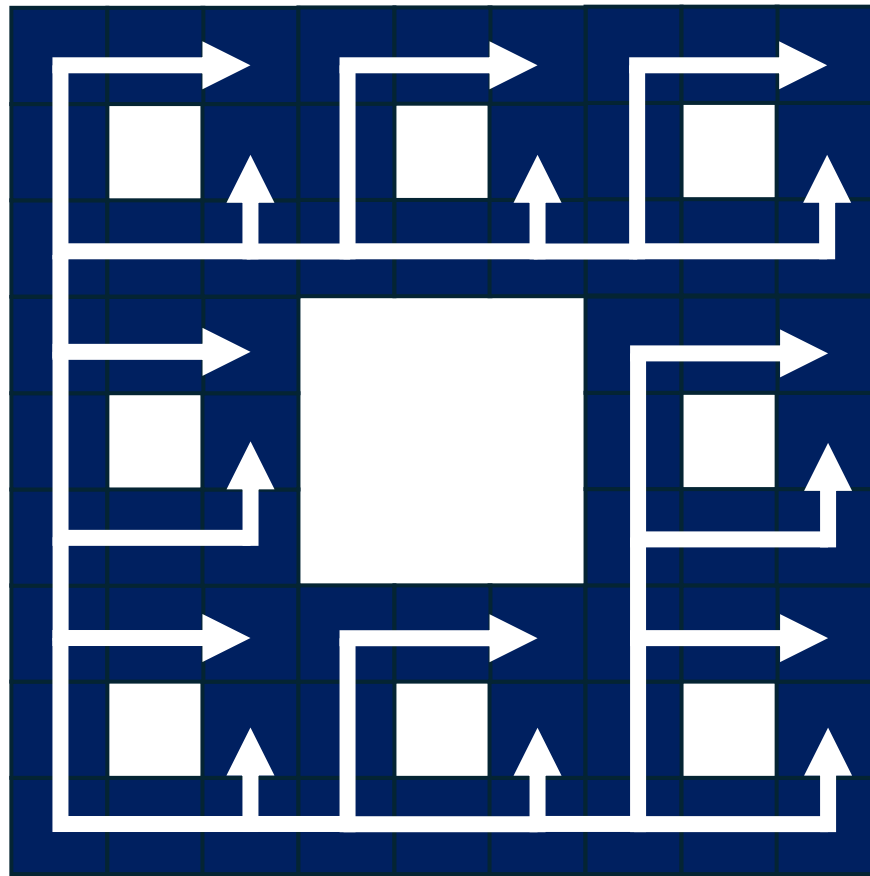
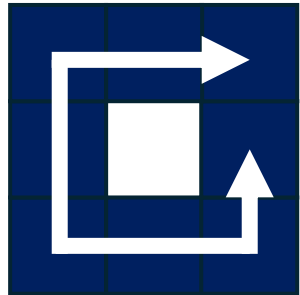
Step 4: Resetting



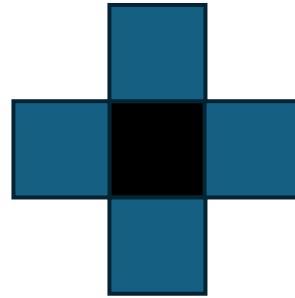
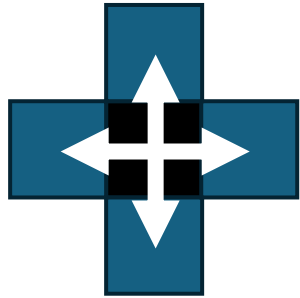
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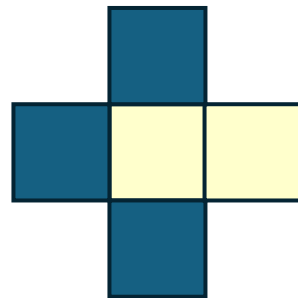
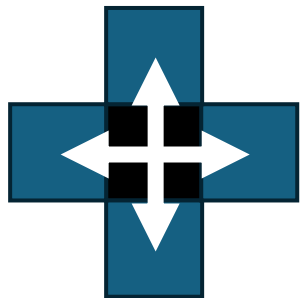
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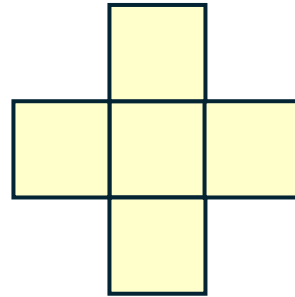
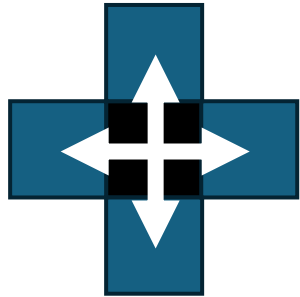
Synchronization



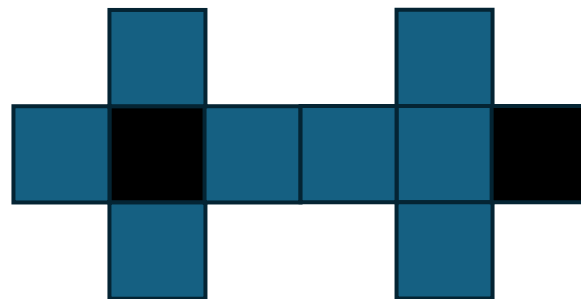
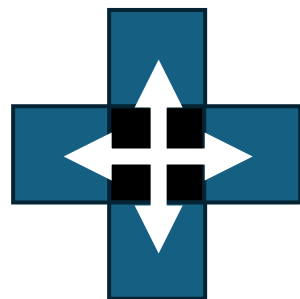
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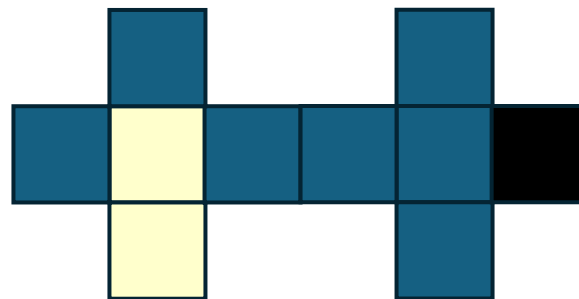
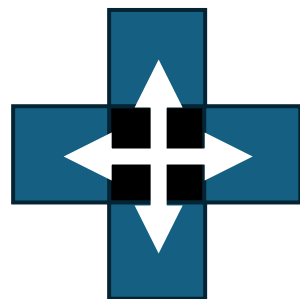
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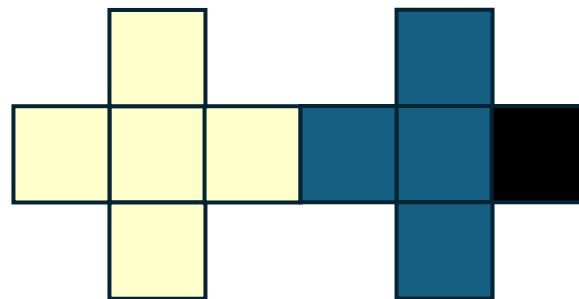
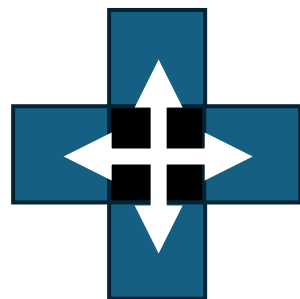
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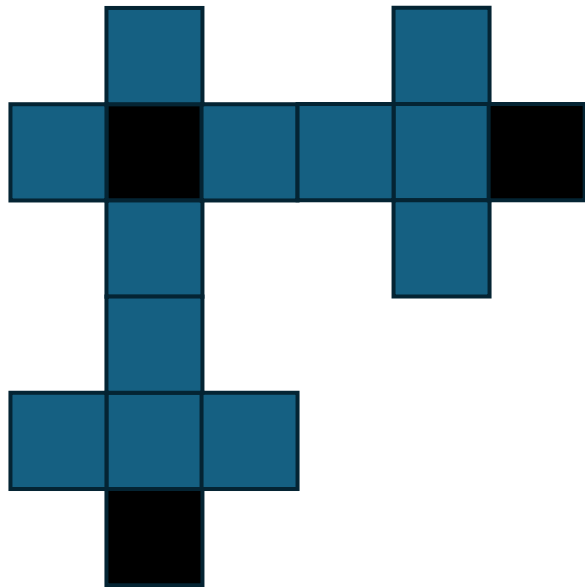
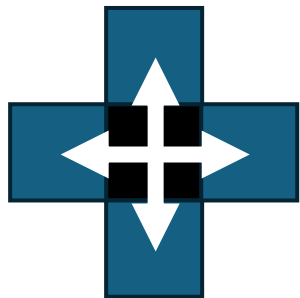
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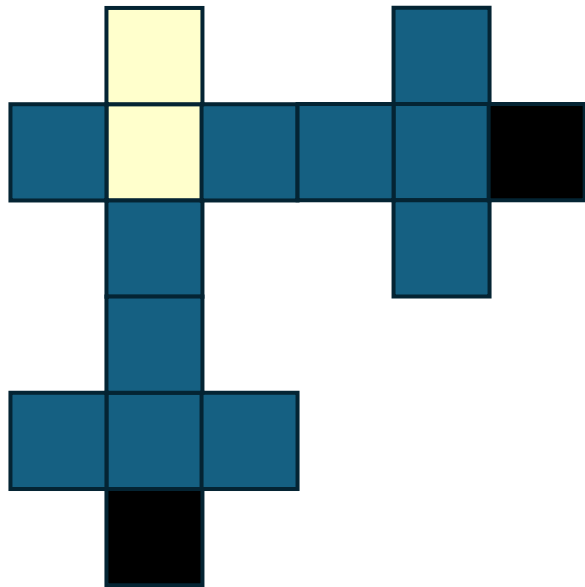
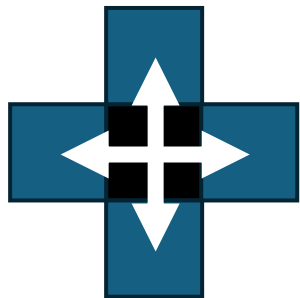
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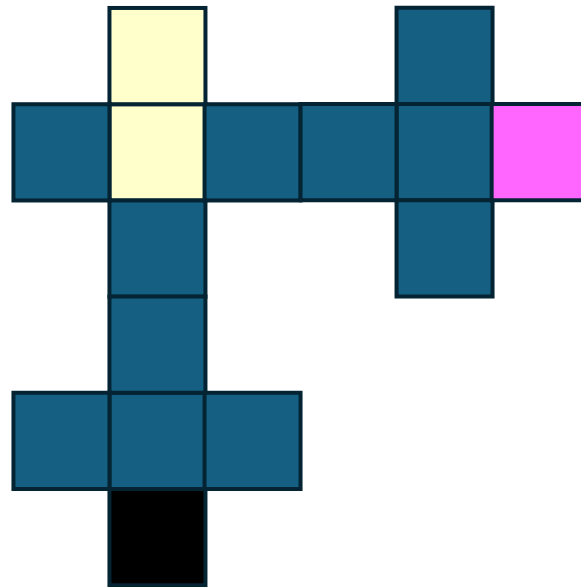
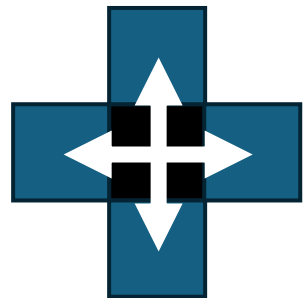
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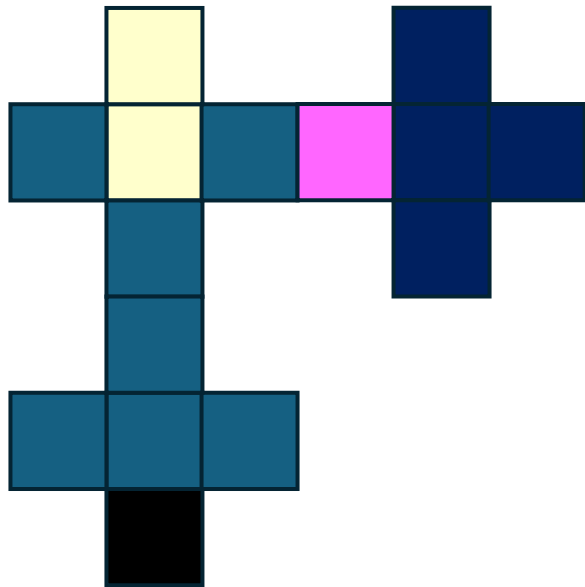
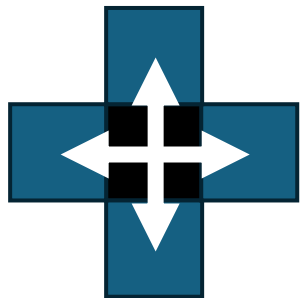
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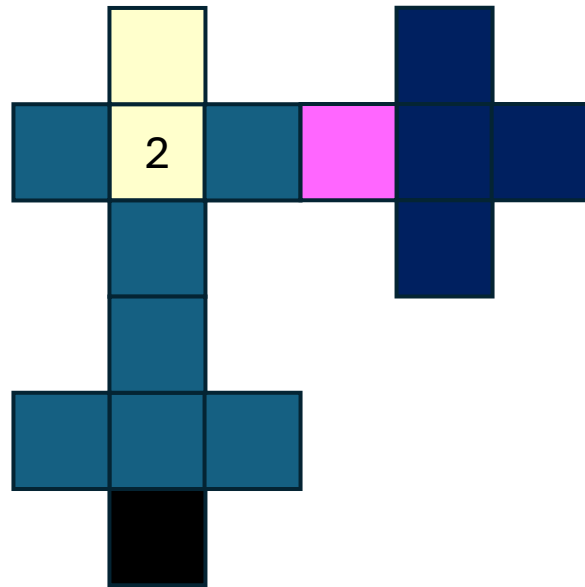
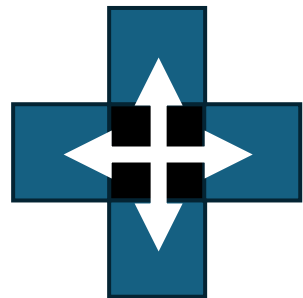
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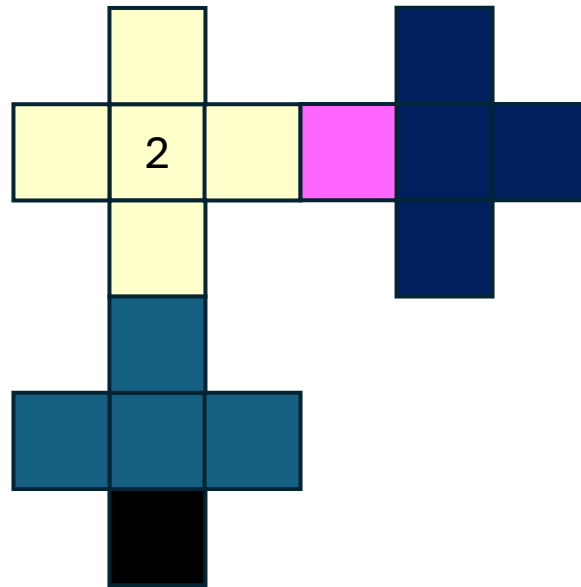
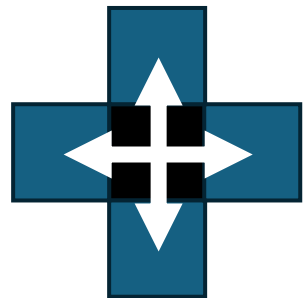
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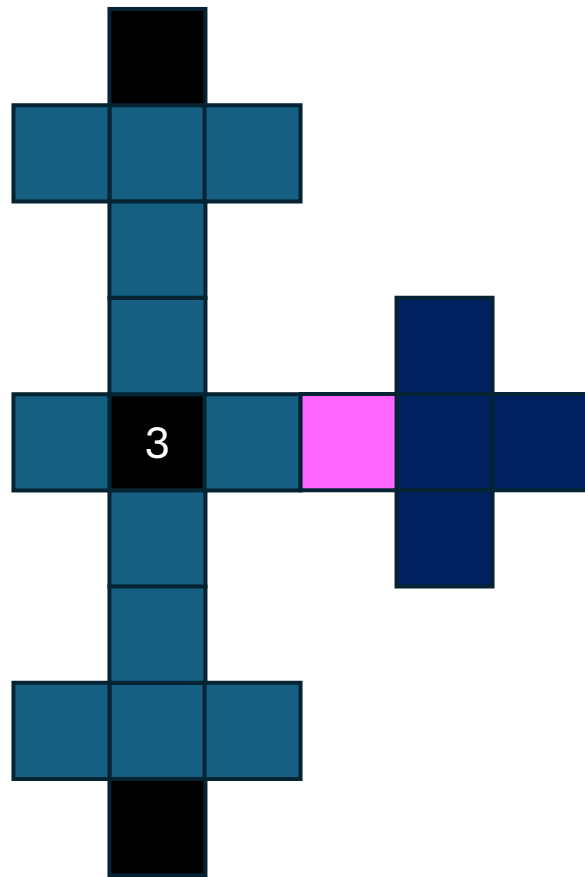
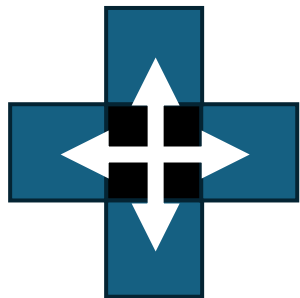
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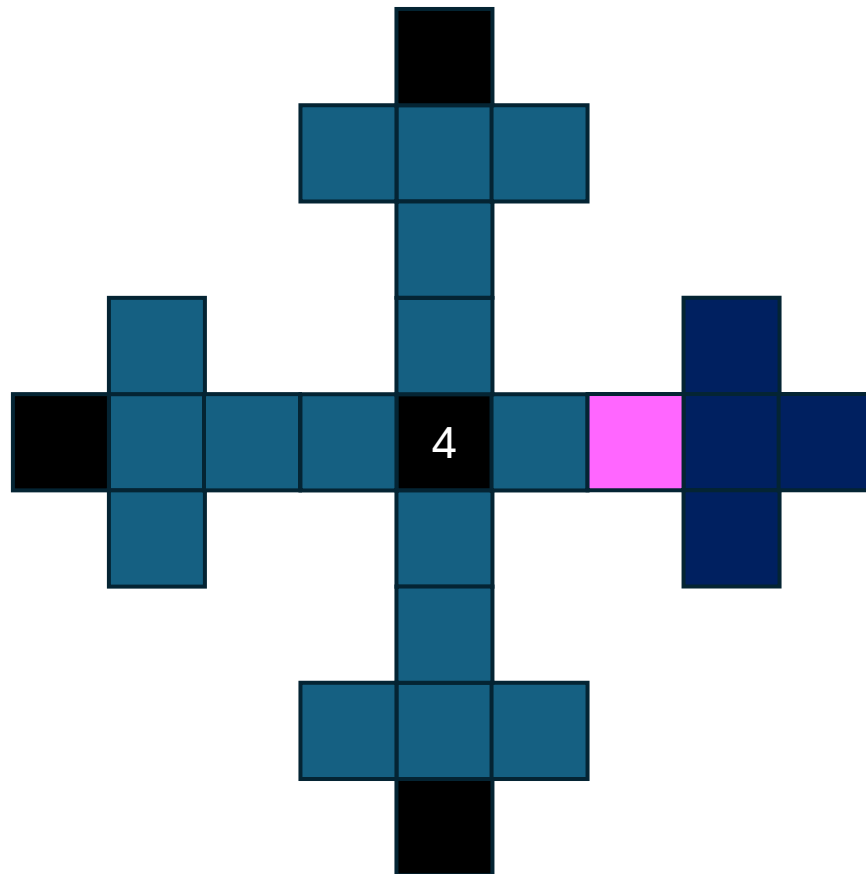
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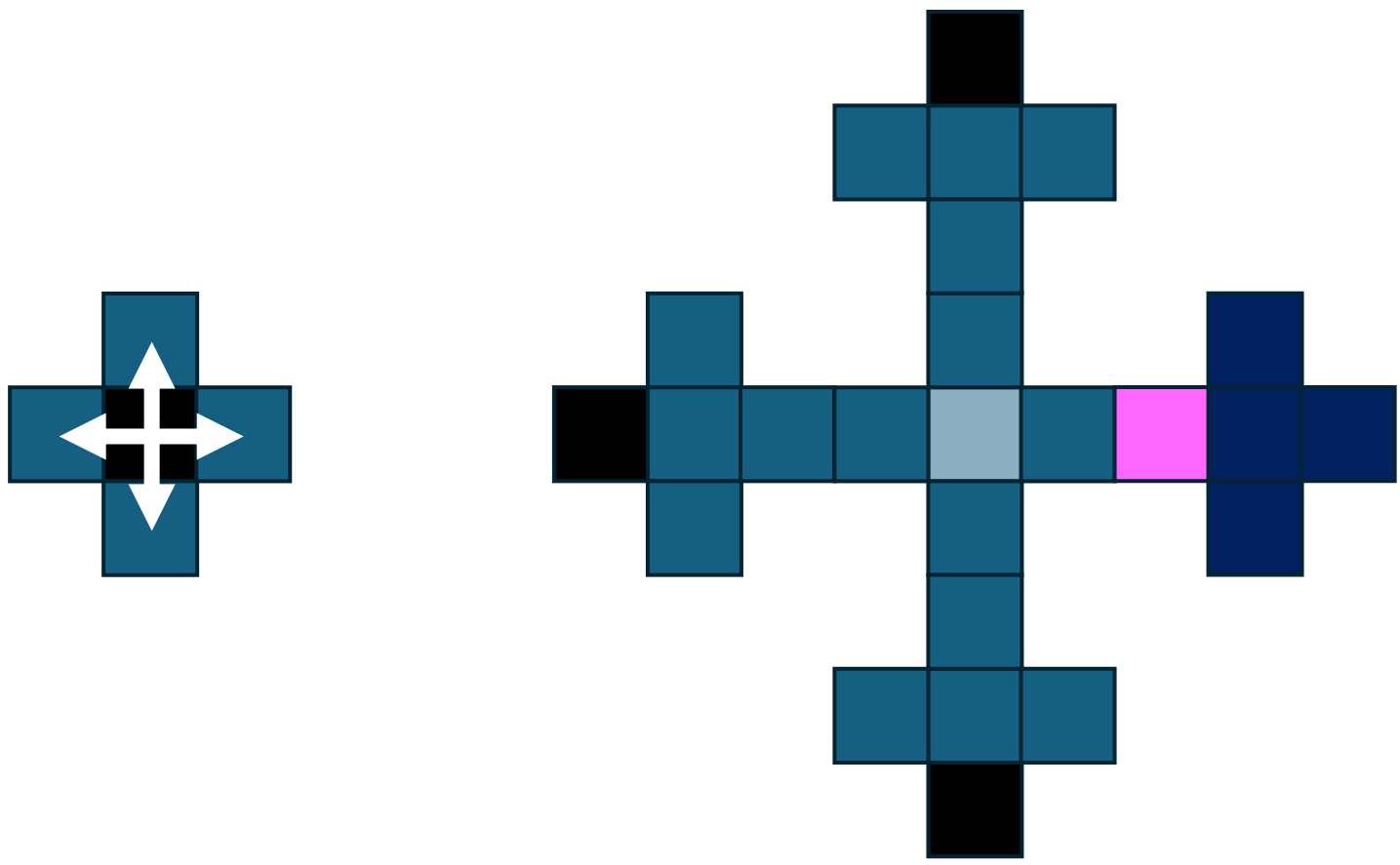
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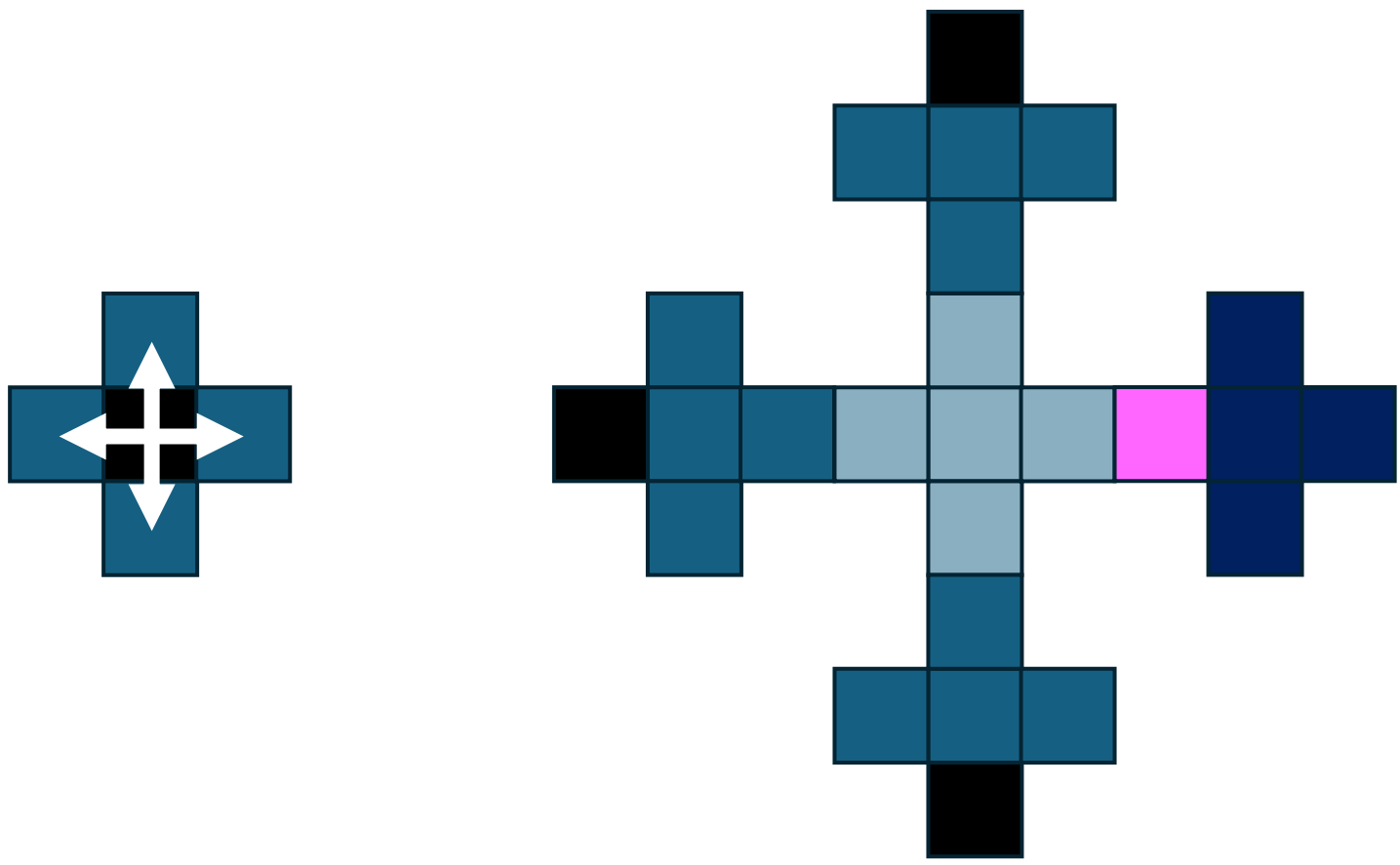
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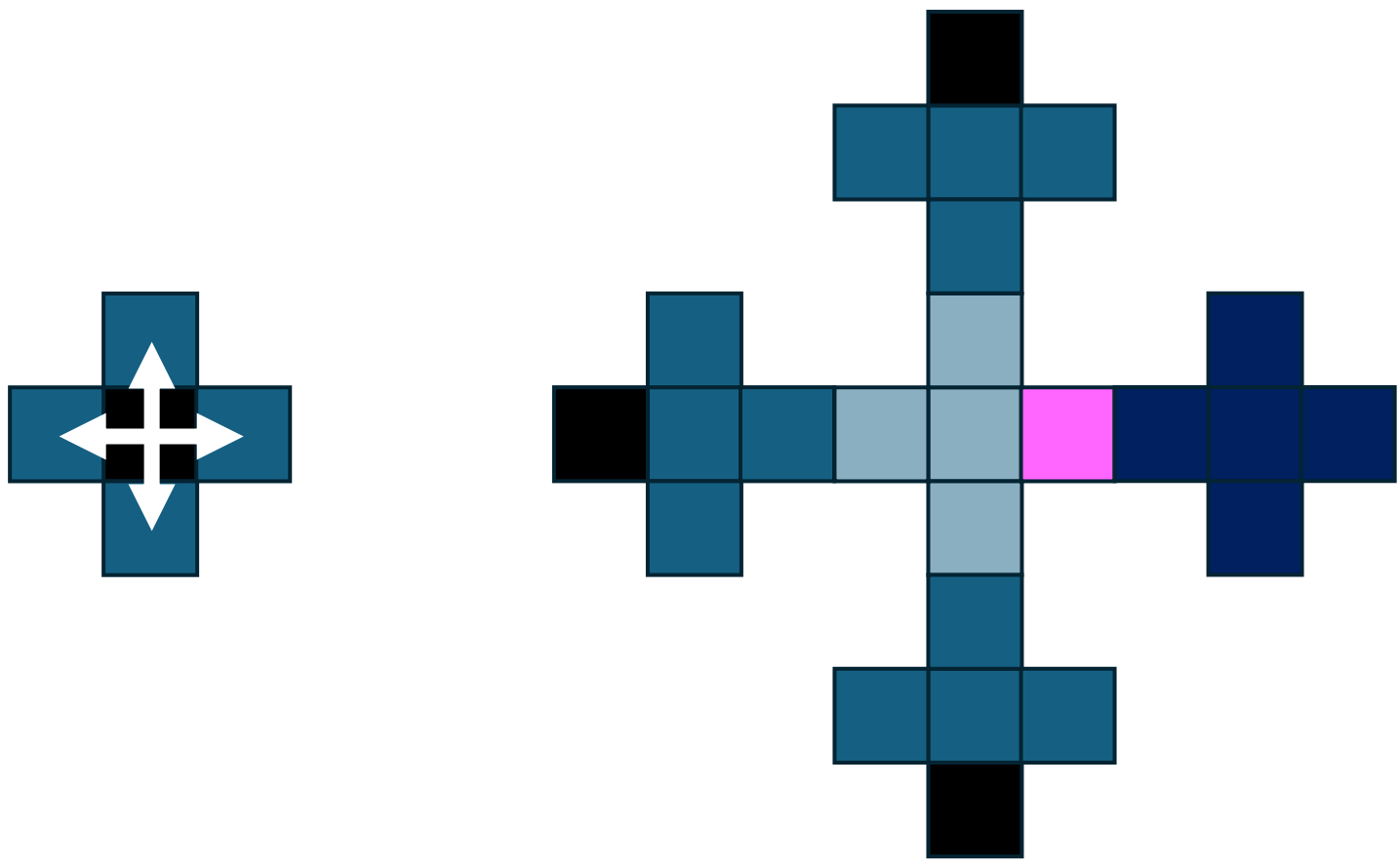
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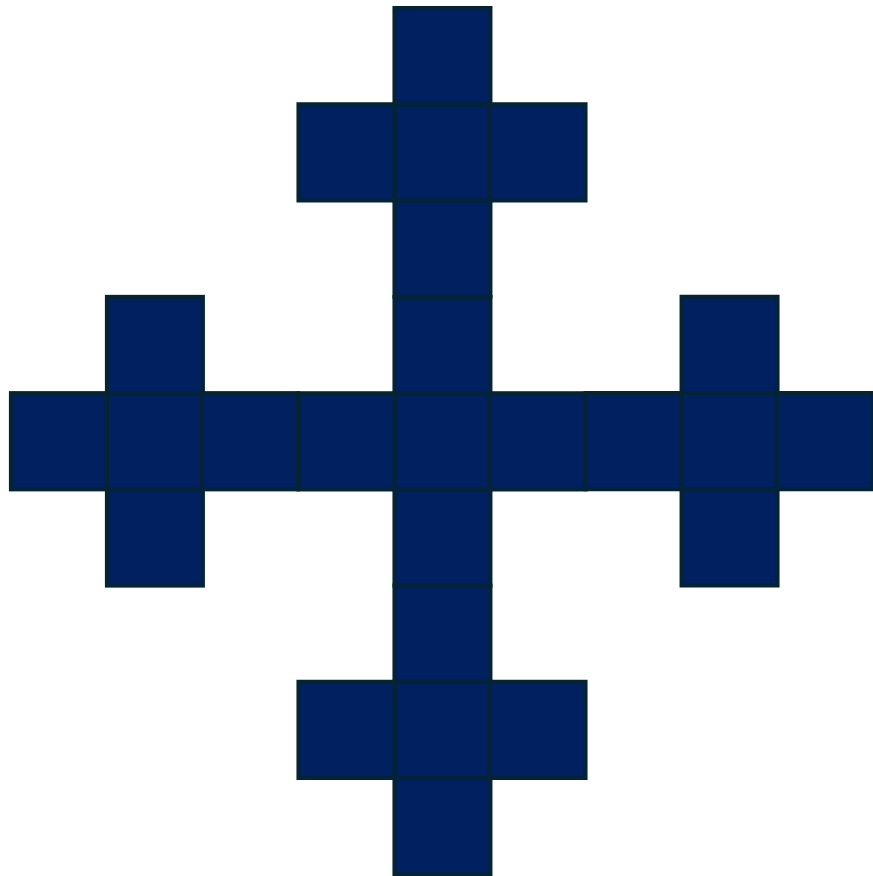
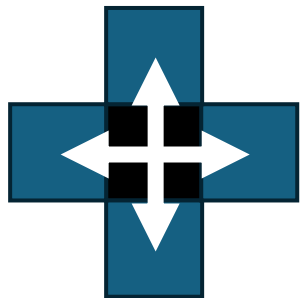
Synchronization



Synchronization



Synchronization



Final Result (Sierpinski Triangle)

Conclusion

What is left?

Conclusion

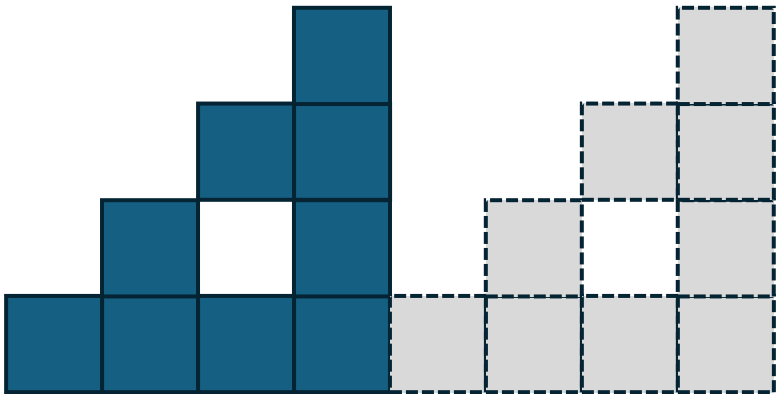
- All fractals are buildable at temperature 1
- A single system exists that builds all these fractals
- Do higher temperatures allow for more efficient fractal constructions?
- Are there special classes of fractals which can be built with small systems?

Fractals in Seeded TA

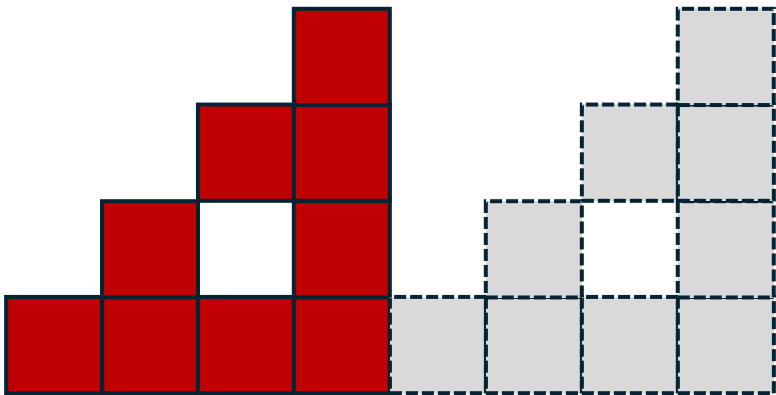


Authors: Asher Haun, **Ryan Knobel**, Adrian Salinas, Ramiro Santos,
Robert Schweller, Timothy Wylie

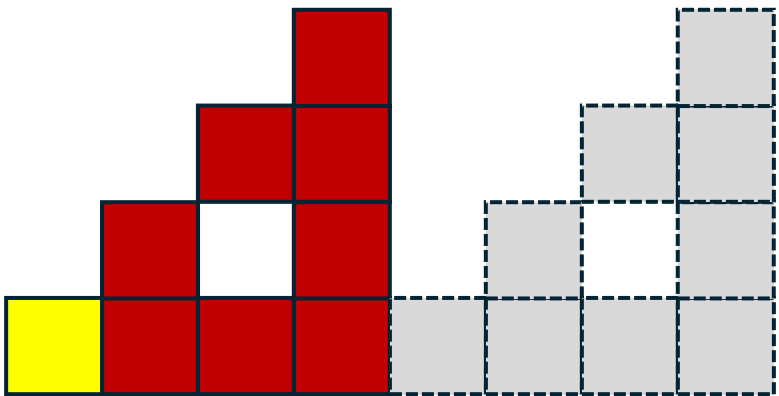
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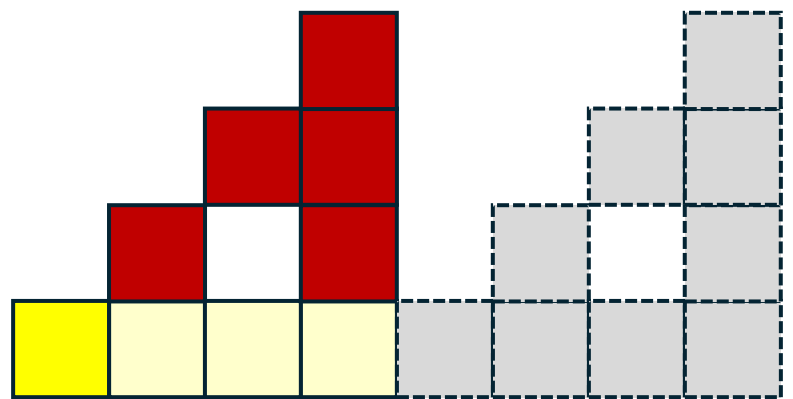
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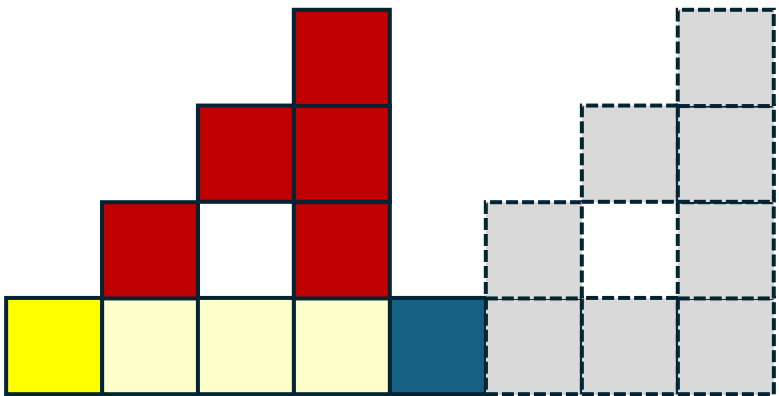
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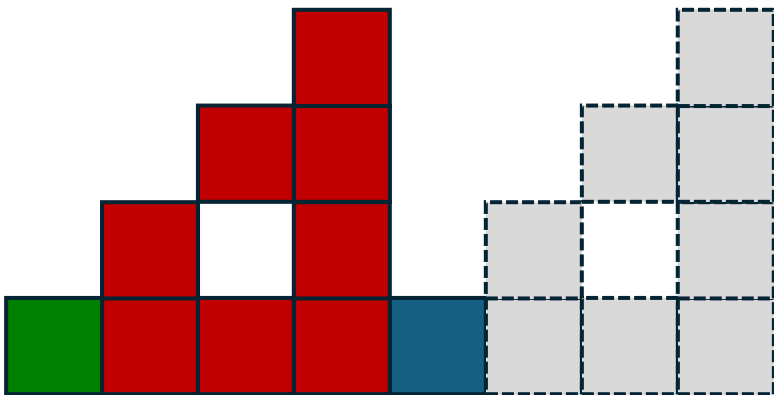
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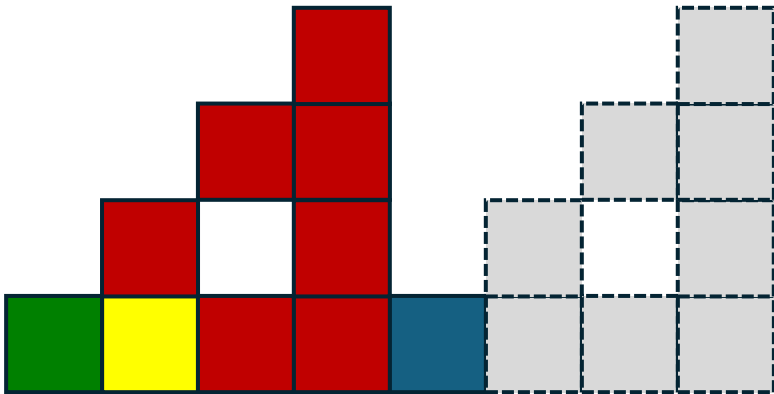
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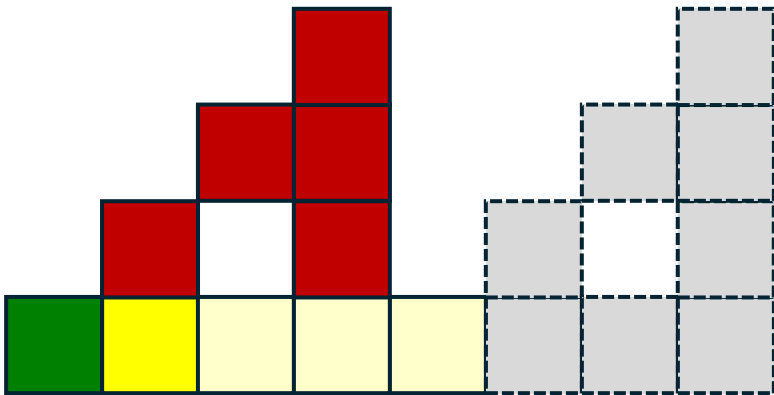
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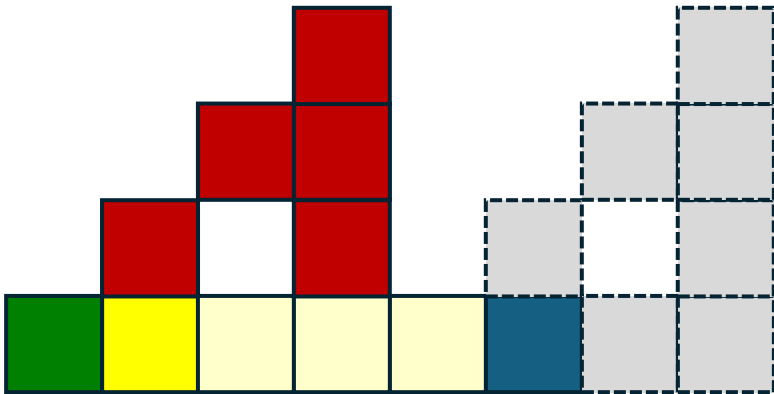
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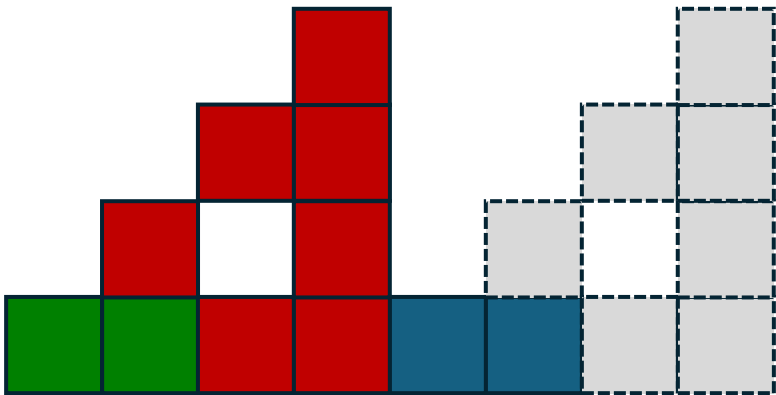
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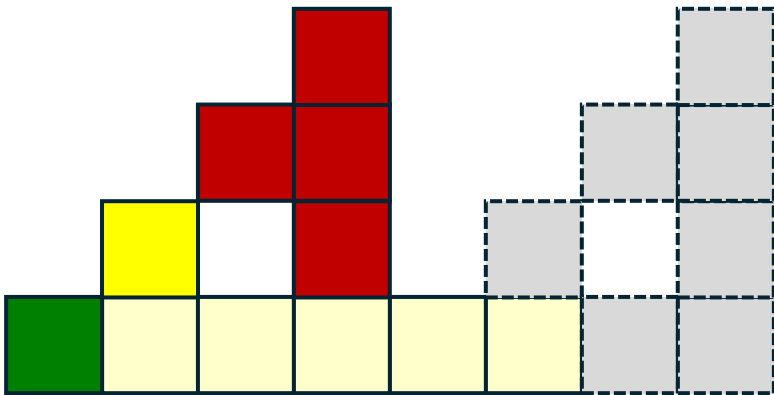
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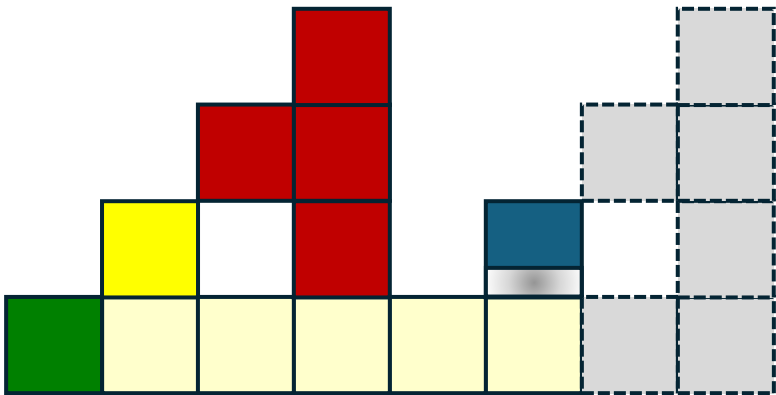
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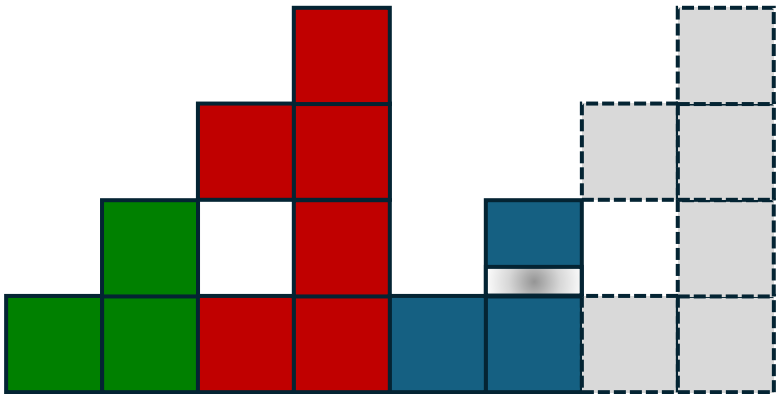
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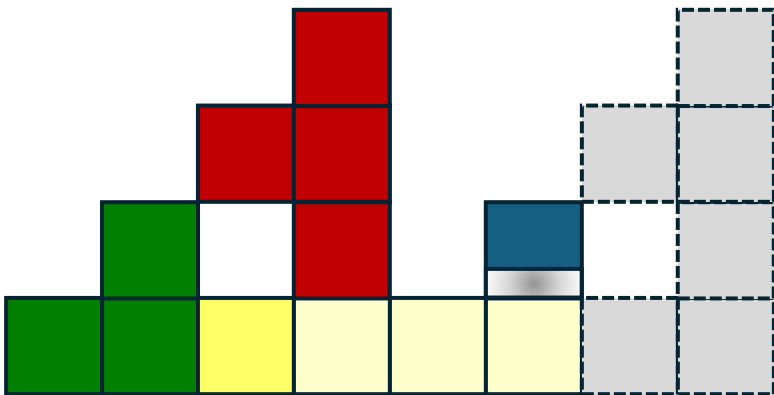
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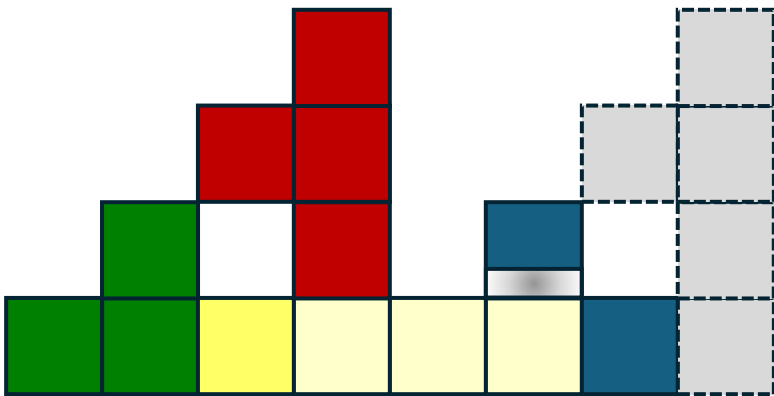
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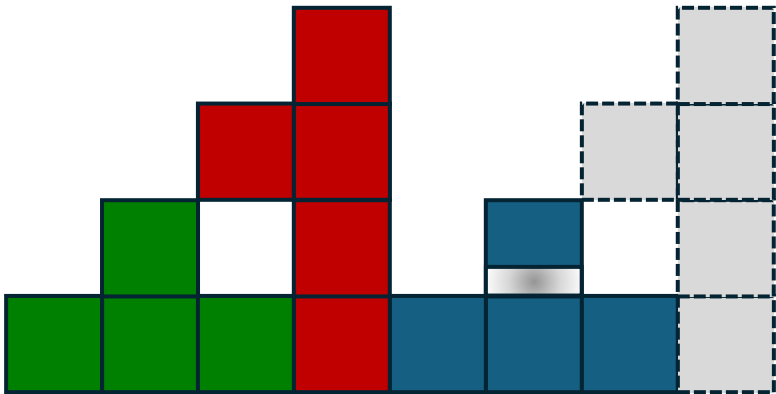
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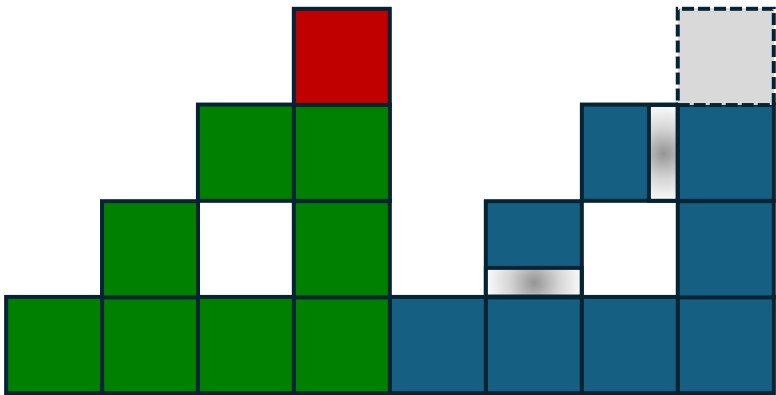
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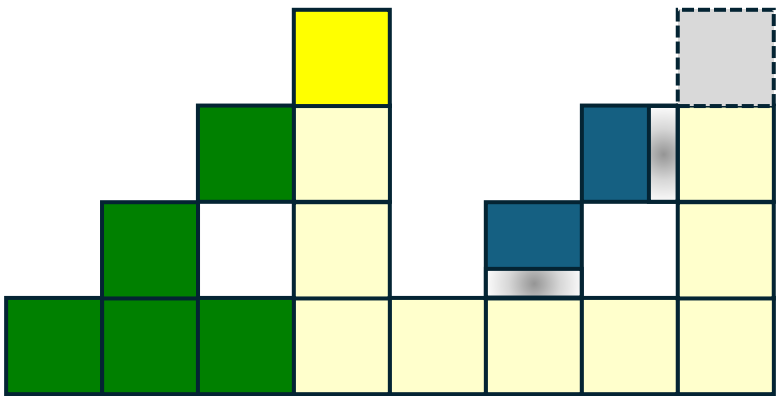
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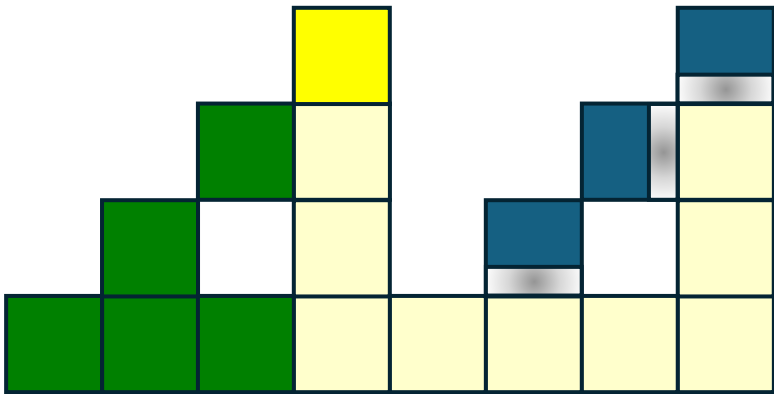
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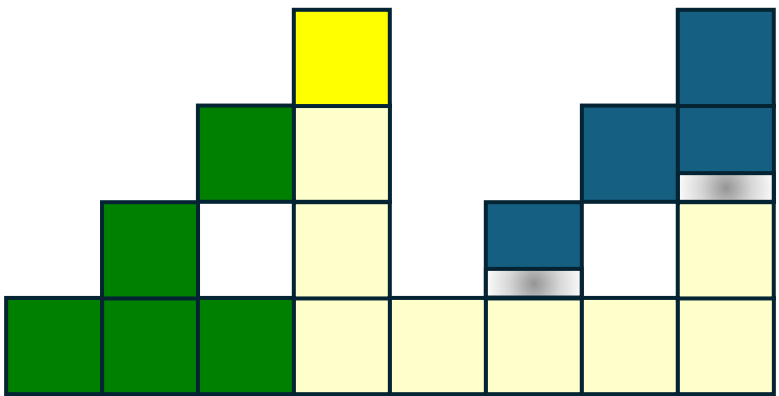
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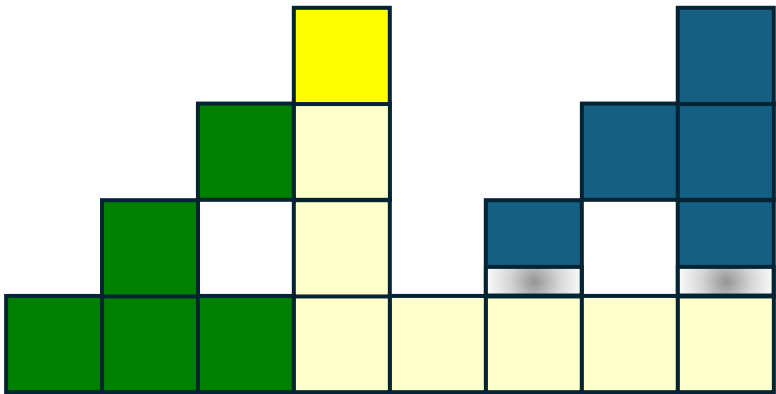
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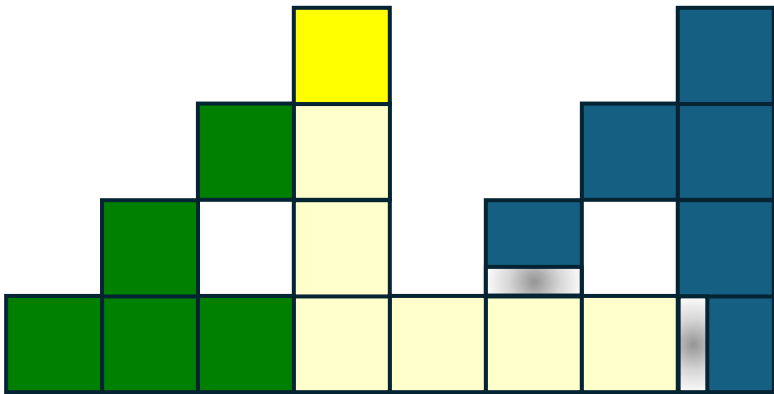
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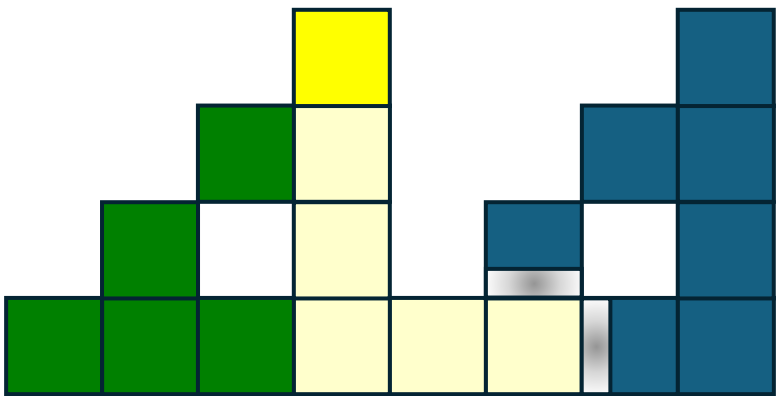
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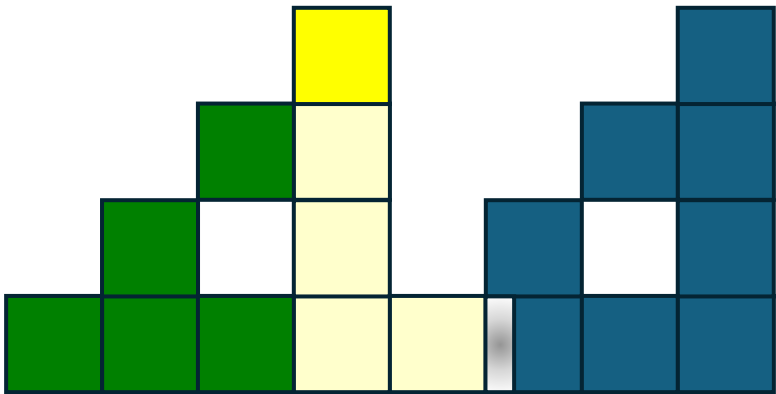
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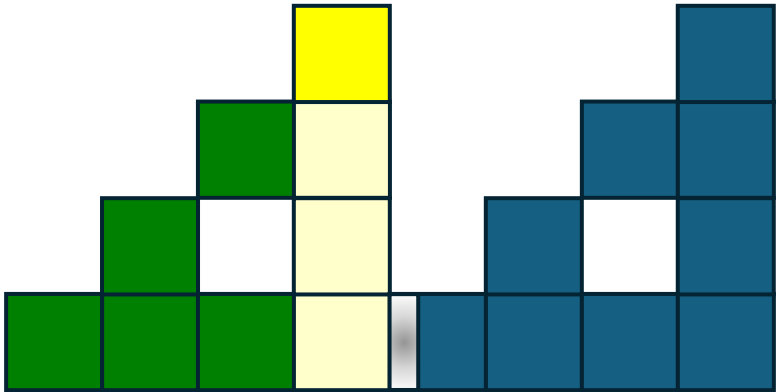
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Step 1: A Copying Procedure

